Slabs and Flat Slabs

Lecture 5
19th October 2016

Contents - Lecture 5

- Designing for shear in slabs - including punching shear
- Detailing - Solid slabs
- Flat Slab Design - includes flexure worked example
- Exercise - Punching shear
Designing for shear in slabs

- When shear reinforcement is not required - e.g. usually one and two-way spanning slabs
- Punching shear - e.g. flat slabs and pad foundations

Shear

There are three approaches to designing for shear:
- When shear reinforcement is not required e.g. usually slabs
- When shear reinforcement is required e.g. Beams, see Lecture 3
- Punching shear requirements e.g. flat slabs

The maximum shear strength in the UK should not exceed that of class C50/60 concrete
Shear resistance without shear reinforcement

\[ V_{Rd,c} = [0.12k(100 \rho_1 f_{ck})^{1/3} + 0.15 \sigma_{cp}] b_w d \]  \hspace{1cm} (6.2.a)

with a minimum of
\[ V_{Rd,c} = (0.035k^{3/2}f_{ck}^{1/2} + 0.15 \sigma_{cp}) b_w d \]  \hspace{1cm} (6.2.b)

where:
- \( k = 1 + \sqrt{200/d} \leq 2.0 \)
- \( \rho = A_{sl}/b_w d \leq 0.02 \)
- \( A_{sl} \) = area of the tensile reinforcement,
- \( b_w \) = smallest width of the cross-section in the tensile area [mm]
- \( \sigma_{cp} = N_{Ed}/A_c < 0.2 f_{cs} \) [MPa]  \hspace{1cm} Compression +ve
- \( N_{Ed} \) = axial force in the cross-section due to loading or pre-stressing [in N]
- \( A_c \) = area of concrete cross section [mm\(^2\)]
**Shear - \( v_{Rd,c} \)**

**Concise Table 7.1 or 15.6**

<table>
<thead>
<tr>
<th>Effective depth, ( d ) (mm)</th>
<th>( A_s (bd) % )</th>
<th>( \leq 200)</th>
<th>( 225)</th>
<th>( 250)</th>
<th>( 275)</th>
<th>( 300)</th>
<th>( 350)</th>
<th>( 400)</th>
<th>( 450)</th>
<th>( 500)</th>
<th>( 600)</th>
<th>( 750)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.25 )</td>
<td>0.54</td>
<td>0.52</td>
<td>0.50</td>
<td>0.48</td>
<td>0.47</td>
<td>0.45</td>
<td>0.43</td>
<td>0.41</td>
<td>0.40</td>
<td>0.38</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>( 0.50 )</td>
<td>0.57</td>
<td>0.57</td>
<td>0.56</td>
<td>0.55</td>
<td>0.54</td>
<td>0.52</td>
<td>0.51</td>
<td>0.49</td>
<td>0.48</td>
<td>0.47</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>( 0.75 )</td>
<td>0.68</td>
<td>0.66</td>
<td>0.64</td>
<td>0.63</td>
<td>0.62</td>
<td>0.59</td>
<td>0.58</td>
<td>0.56</td>
<td>0.55</td>
<td>0.53</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>( 1.00 )</td>
<td>0.75</td>
<td>0.72</td>
<td>0.71</td>
<td>0.69</td>
<td>0.68</td>
<td>0.65</td>
<td>0.64</td>
<td>0.62</td>
<td>0.61</td>
<td>0.59</td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td>( 1.25 )</td>
<td>0.80</td>
<td>0.78</td>
<td>0.76</td>
<td>0.74</td>
<td>0.73</td>
<td>0.71</td>
<td>0.69</td>
<td>0.67</td>
<td>0.66</td>
<td>0.63</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>( 1.50 )</td>
<td>0.85</td>
<td>0.83</td>
<td>0.81</td>
<td>0.79</td>
<td>0.78</td>
<td>0.75</td>
<td>0.73</td>
<td>0.71</td>
<td>0.70</td>
<td>0.67</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>( 1.75 )</td>
<td>0.90</td>
<td>0.87</td>
<td>0.85</td>
<td>0.83</td>
<td>0.82</td>
<td>0.79</td>
<td>0.77</td>
<td>0.75</td>
<td>0.73</td>
<td>0.71</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>( 2.00 )</td>
<td>0.94</td>
<td>0.91</td>
<td>0.89</td>
<td>0.87</td>
<td>0.85</td>
<td>0.82</td>
<td>0.80</td>
<td>0.78</td>
<td>0.77</td>
<td>0.74</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>( k )</td>
<td>2.00</td>
<td>1.94</td>
<td>1.89</td>
<td>1.85</td>
<td>1.82</td>
<td>1.76</td>
<td>1.71</td>
<td>1.67</td>
<td>1.63</td>
<td>1.58</td>
<td>1.52</td>
<td></td>
</tr>
</tbody>
</table>

Table derived from: \( \bar{V}_{Rd,c} = 0.12 k (100 \rho I f_{ck})^{1/3} \geq 0.035 k^{1.5} f_{ck}^{0.5} \) where \( k = 1 + \sqrt{(200/d)} \leq 2 \) and \( \rho = A_s (bd) / 0.02 \)

Note: This table has been prepared for \( f_{ck} = 30 \). Where \( \rho \) exceeds 0.40%, the following factors may be used :

- \( f_{ck} \) factor 25 28 32 35 40 45 50
- 0.94 0.98 1.02 1.05 1.10 1.14 1.19

**Shear in Slabs**

Most slabs do not require shear reinforcement

∴ Check \( V_{Ed} < V_{Rd,c} \)

Where \( V_{Rd,c} \) is shear resistance of members without reinforcement

\[
V_{Rd,c} = 0.12 k (100 \rho I f_{ck})^{1/3} \geq 0.035 k^{1.5} f_{ck}^{0.5}
\]

Where \( V_{Ed} > V_{Rd,c} \), shear reinforcement is required and the strut inclination method should be used

How-to Compendium p21
Punching shear

Punching shear symbols

\[ u_i = i^{th} \text{ perimeter} \]
\[ u_i^c = \text{basic control perimeter at } 2d \]
\[ u_{i,r} = \text{reduced basic control perimeter} \]
\[ u_0 = \text{column perimeter} \]
\[ d = \text{average effective depth} \]
\[ k = \text{coeff. depending on column shape - see Table 6.1} \]
\[ W_i = \text{a shear distribution factor - see 6.4.3(3)} \]

Punching shear does not use the Variable Strut inclination method and is similar to BS 8110 methods

- The basic control perimeter is set at \( 2d \) from the loaded area
- The shape of control perimeters have rounded corners

- Where shear reinforcement is required the shear resistance is the sum of the concrete and shear reinforcement resistances.
Punching Shear

EC2: Cl. 6.4.3 & 6.4.4

6.4.3 (2)

(b) Punching shear reinforcement is not necessary if:

$$\frac{v_{Ed}}{V_{Rd,c}} \leq 0.02$$

When calculating $$v_{Rd,c}$$:

6.4.4 (1)

$$\rho = \sqrt{\rho_y \cdot \rho_z} \leq 0.02$$

$$\rho_y, \rho_z$$ relate to the bonded tension steel in y- and z- directions respectively. The values $$\rho_y$$ and $$\rho_z$$ should be calculated as mean values taking into account a slab width equal to the column width plus 3d each side.

Punching Shear

The applied shear stress should be taken as:

$$v_{Ed} = \beta V_{Ed}/u_1 \cdot d$$ where $$\beta = 1 + k(M_{Ed}/V_{Ed})u_1/W_1$$

For structures where:

- lateral stability does not depend on frame action
- adjacent spans do not differ by more than 25%

the approximate values for $$\beta$$ shown may be used:
Punching Shear

Where the simplified arrangement is not applicable then $\beta$ can be calculated

For a rectangular internal column with biaxial bending the following simplification may be used:

$$\beta = 1 + 1.8\left\{ \left( \frac{e_y}{b_y} \right)^2 + \left( \frac{e_z}{b_z} \right)^2 \right\}^{0.5}$$

where $b_y$ and $b_z$ are the dimensions of the control perimeter

For other situations there is plenty of guidance on determining $\beta$ given in Cl 6.4.3 of the Code.

Punching shear control perimeters

**Basic perimeter, $u_1$**

EC2: Cl. 6.4.2

Near to an edge

Concise: Figure 8.4

Near to an opening

Concise: Figure 8.6
The outer control perimeter at which shear reinforcement is not required, should be calculated from:

\[ u_{\text{out,ef}} = \frac{\beta V_d}{v_{Rd,c} d} \]

The outermost perimeter of shear reinforcement should be placed at a distance not greater than \( kd \) (\( k = 1.5 \)) within the outer control perimeter.

Where proprietary systems are used the control perimeter at which shear reinforcement is not required, \( u_{\text{out}} \) or \( u_{\text{out,ef}} \) (see Figure) should be calculated from the following expression:

\[ u_{\text{out,ef}} = \frac{8 V_d}{v_{Rd,c} d} \]
Punching Shear Reinforcement

EC 2: Cl. 6.4.5, Equ. 6.52 Concise: 8.5

Where shear reinforcement is required it should be calculated in accordance with the following expression:

\[ v_{rd,cs} = 0.75 \cdot v_{rd,c} + 1.5 \cdot (d/s_r) \cdot A_{sw} \cdot f_{ywd,ef} \cdot (1/(u_1d)) \cdot \sin \alpha \]  

(6.52)

- \( A_{sw} \) = area of shear reinforcement in each perimeter around the column.
- \( s_r \) = radial spacing of layers of shear reinforcement
- \( \alpha \) = angle between the shear reinforcement and the plane of slab
- \( f_{ywd,ef} \) = effective design strength of the punching shear reinforcement, \( \leq 250 + 0.25 \cdot d \leq f_{ywd} \) (MPa.)
- \( d \) = mean effective depth of the slabs (mm)

Max. shear stress at column face,

\[ v_{Ed} = \frac{\beta V_{Ed}}{u_0d} \leq v_{rd,max} = 0.5 \cdot f_{cd} \]

EC2 Equ. 6.53

---

Punching Shear Reinforcement

EC 2: Cl. 6.4.5 (3), Equ. 6.53 Concise: 8.6

Max. shear stress at column face, the \( u_0 \) perimeter

\[ v_{Ed} = \frac{\beta V_{Ed}}{u_0d} \leq v_{rd,max} = 0.5 \cdot f_{cd} \]

where

- \( \beta \) = factor dealing with eccentricity (see Table 6.4)
- \( V_{Ed} \) = applied shear force
- \( d \) = mean effective depth
- \( u_0 \) =
  - \( 2(c_1 + c_2) \) for interior columns
  - \( c_2 + 3d \leq c_2 + 2c_1 \) for edge columns
  - \( 3d \leq c_2 + 2c_1 \) for corner columns

where

- \( c_1 \) = column depth
- \( c_2 \) = column width

\( c_1 \) and \( c_2 \) are illustrated in Concise Figure 8.5
Punching Shear Reinforcement

Check $v_{Ed} \leq 2 \check{v}_{Rdc}$ at basic control perimeter \( \text{ (NA check)} \)

Note: UK NA says ‘first’ control perimeter, but the paper* on which this guidance is based says ‘basic’ control perimeter

The minimum area of a link leg (or equivalent), $A_{sw,\text{min}}$, is given by the following expression:

$$A_{sw,\text{min}} \geq \frac{(1.5 \sin \alpha + \cos \alpha) \cdot (s_t, s_i)}{0.08 \sqrt{f_{ck}}} / f_{yk}$$  \( \text{EC2 equ 9.11} \)

$$A_{sw,\text{min}} \geq \frac{(0.053 s_t, s_i) \sqrt{(f_{ck})}}{f_{yk}} \quad \text{For vertical links}$$


Punching shear Worked example

From Worked Examples to EC2: Volume 1
Example 3.4.10
Punching shear at column C2

- At C2 the ultimate column reaction is 1204.8 kN
- Effective depths are 260mm & 240mm
- Reinforcement: $\rho_{ly} = 0.0085$, $\rho_{lz} = 0.0048$

Design information

- 400 mm Square Column
- 300 mm flat slab C30/37 concrete

A few definitions:

- $u_0$
- $u_i$
- $u_{out}$
- Outer control perimeter requiring shear reinforcement, $u_{int}$
- Outer control perimeter not requiring shear reinforcement, $u_{out}$
Solution

1. Check shear at the perimeter of the column

\[ V_{Ed} = \beta V_{Ed} / (U_0 d) < V_{Rd,max} \]

\[ \beta = 1.15 \]

\[ U_0 = 4 \times 400 = 1600 \text{ mm} \]

\[ d = (260 + 240)/2 = 250 \text{ mm} \]

\[ V_{Ed} = 1.15 \times 1204.8 \times 1000 / (1600 \times 250) = 3.46 \text{ MPa} \]

\[ V_{Rd,max} = 0.5 \nu f_{cd} \]

\[ = 0.5 \times 0.6(1-f_{ck}/250) \times \alpha cc f_{ck} / \gamma m \]

\[ = 0.5 \times 0.6(1-30/250) \times 1.0 \times 30 / 1.5 = 5.28 \text{ MPa} \]

\[ V_{Ed} < V_{Rd,max} \quad \ldots \text{OK} \]

Solution

2. Check shear at \( u_1 \), the basic control perimeter

\[ V_{Ed} = \beta V_{Ed} / (U_1 d) < V_{Rd,c} \]

\[ \beta = \text{as before} \]

\[ U_1 = 2(c_x + c_y) + 2\pi x 2d \]

\[ = 2(400 + 400) + 2\pi x 2 \times 250 = 4742 \text{ mm} \]

\[ V_{Ed} = 1.15 \times 1204.8 \times 1000 / (4742 \times 250) = 1.17 \text{ MPa} \]

\[ V_{Rd,c} = 0.12 k(100/\alpha) f_{ch} \]

\[ k = 1 + (200/d)^{1/2} = 1 + (200/250)^{1/2} = 1.89 \]

\[ \alpha = (\rho \lambda) \]

\[ \lambda = (0.0085 \times 0.0048)^{1/2} = 0.0064 \]

\[ V_{Rd,c} = 0.12 \times 1.89(100 \times 0.0064 \times 30)^{1/3} = 0.61 \text{ MPa} \]

\[ V_{Ed} > V_{Rd,c} \]

1.17 MPa > 0.61 MPa \ldots \text{Therefore punching shear reinf. required}

2a. NA check:

\[ V_{Ed} \leq 2V_{Rd,c} \text{ at basic control perimeter} \]

\[ 1.17 \text{ MPa} \leq 2 \times 0.61 \text{ MPa} = 1.22 \text{ MPa} \quad \text{OK} \]
Solution

3. Perimeter at which punching shear no longer required

\[ u_{\text{out}} = \beta V_{Ed} / (d v_{Rd,c}) \]
\[ = 1.15 \times 1204.8 \times 1000 / (250 \times 0.61) \]
\[ = 9085 \text{ mm} \]

Rearrange:

\[ u_{\text{out}} = (u_{\text{out}} - 2(c_x + c_y)) / (2\pi) \]
\[ r_{\text{out}} = (9085 - 1600) / (2\pi) = 1191 \text{ mm} \]

Position of outer perimeter of reinforcement from column face:

\[ r = 1191 - 1.5 \times 250 = 816 \text{ mm} \]

Maximum radial spacing of reinforcement:

\[ s_{r,\text{max}} = 0.75 \times 250 = 187 \text{ mm, say 175 mm} \]

Solution

4. Area of reinforcement

\[ A_{sw} \geq (V_{Ed} - 0.75V_{Rd,c})s_t / (1.5f_{yd,ef}) \]
\[ f_{yd,ef} = (250 + 0.25d) = 312 \text{ MPa} \]
\[ A_{sw} \geq (1.17 - 0.75 \times 0.61) \times 175 \times 4741 / (1.5 \times 312) \]
\[ \geq 1263 \text{ mm}^2 / \text{perim.} \]

Minimum area of a link leg:

\[ A_{sw,\text{min}} \geq (0.053 s, s_t \sqrt{f_{ck}}) / f_{yk} = 0.053 \times 175 \times 350 \times 30 / 500 \]
\[ \geq 36 \text{ mm}^2 \]

1H10 is 78.5 mm² dia.  
16H10 = 1256 mm² / perim  
See layout on next slide  
16H10 = 1256 mm² / perim  
See layout on next slide  
could use H8s (50 mm²) but would need 26 per perimeter So use H10s (same price as H8s!)
Solution

Detailing - Solid slabs
**Detailing - Solid slabs**

EC2: Cl. 9.3

Rules for one-way and two-way solid slabs

- Generally: as for beams.
- Where partial fixity exists, but not taken into account in design:
  - Internal supports: $A_{s,top} \geq 0.25A_s$ for $M_{\text{max}}$ in adjacent span
  - End supports: $A_{s,top} \geq 0.15A_s$ for $M_{\text{max}}$ in adjacent span
- This top reinforcement should extend $\geq 0.2$ adjacent span
- Reinforcement at free edges should include ‘u’ bars and longitudinal bars

$\begin{align*}
\text{Diagram:} & \quad h \\
\text{Minimum clear span:} & \quad \geq 2h
\end{align*}$

- Secondary reinforcement 20%

---

**Flat Slab Design**
Flat Slab Design - Contents

Flat slabs - Introduction

EC2 particular rules for flat slabs

Initial sizing

Analysis methods - BM’s and Shear Force

Design constraints
  - Punching shear
  - Deflection
  - Moment transfer from slab to column

Flat Slabs - Introduction

What are flat slabs?
  - Solid concrete floors of constant thickness
  - They have flat soffits
Flat Slabs - Introduction

1. COBIAX
2. BUBBLEDECK
Poll Q1:
Shear resistance of a beam section

What is the shear resistance governed by the crushing of compression struts?

a. $V_{Ed}$
b. $V_{Rd,c}$
c. $V_{Rd,max}$
d. $V_{Rd,s}$

Particular rules for flat slabs
EC2 sections relevant to Flat Slabs:

- Section 6 Ultimate Limit States
  - cl 6.4 Punching (shear) & PD 6687 cl 2.16, 2.17 & 2.1.8
- Section 9 Detailing of members and particular rules
  - Cl 9.4 Flat slabs
    9.4.1 Slab at internal columns
    9.4.2 Slab at edge and corner columns
    9.4.3 Punching shear reinforcement
- Annex I (Informative) Analysis of flat slabs and shear walls
  I.1 Flat Slabs
    I.1.1 General
    I.1.2 Equivalent frame analysis
    I.1.3 Irregular column layout

The Concrete Society, Technical Report 64 - Guide to the Design and Construction of Reinforced Concrete Flat Slabs
Particular rules for flat slabs
Distribution of moments

EC2: Figure I.1

Concise Figure 5.11

Particular rules for flat slabs
Distribution of moments

EC2: Table I.1

Concise: Table 5.2

<table>
<thead>
<tr>
<th>Location</th>
<th>Negative moments</th>
<th>Positive moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column strip</td>
<td>60% – 80%</td>
<td>50% – 70%</td>
</tr>
<tr>
<td>Middle strip</td>
<td>40% – 20%</td>
<td>50% – 30%</td>
</tr>
</tbody>
</table>

Notes
The total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%.
The distribution of design moments given in BS 8110 (column strip: hogging 75%, sagging 55%; middle strip: hogging 25%, sagging 45%) may be used.
Particular rules for flat slabs

EC2: Cl. 9.4
Concise: 12.4.1

• Arrangement of reinforcement should reflect behaviour under working conditions.

• At internal columns $0.5A_t$ should be placed in a width $= 0.25 \times$ panel width.

• At least two bottom bars should pass through internal columns in each orthogonal directions.

Particular rules for flat slabs

EC2: Figure 9.9, I.1.2(5)
Concise Figure 5.12

• Design reinforcement at edge and corner reinforcement should be placed within $b_e$

• The maximum moment that can be transferred from the slab to the column should be limited to $0.17 b_e d f_{ck}$
Moment transfer

Edge and corner columns have limited capacity to transfer moments from slab – redistribution may be necessary

Figure 8

Rebar arrangement
Figure 47

Initial sizing

3 methods:

1. Simple span to depth table
2. Use *Economic Concrete Frame Elements*

<table>
<thead>
<tr>
<th>Imposed Load, Q (kN/m²)</th>
<th>2.5</th>
<th>5</th>
<th>7.5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple Span</td>
<td>28</td>
<td>26</td>
<td>25</td>
<td>23</td>
</tr>
</tbody>
</table>
Initial sizing

3 methods:

1. Simple span to depth table
2. Use Economic Concrete Frame Elements
3. Use Concept.xls

![Diagram]

**Chosen solution:** Flat slab

**Concept:**
- Dimensions: 750 x 750 mm
- Slab thickness: 150 mm
- Reinforcement: 20 mm dia. @ 200 mm

**Project:** Project 1098 - Baseplate for D Valve

**Plan:**
- North is at the top
- Scale: 1:200
- Sections: 3D view

**Materials:**
- Concrete: C25/30
- Steel: S275

**Analysis:**
- Loadings:
  - Dead load
  - Live load
- Connection:
  - Beam-to-column connection

**Details:**
- Reinforcement layout
- Connection details

**Software:** Concept.xls

![Concept.xls Diagram]
Initial sizing

- Elastic Plane Frame - Equivalent Frame Method, Annex I
- Tabular Method - Equivalent Frame Method, Annex I
- Yield Line
  - Plastic method of design
- Finite Element Analysis
  - Elastic method
  - Elasto plastic
Analysis Methods

Elastic Plane Frame - Equivalent Frame Method, Annex I

- Apply in both directions - Y and Z
- Method of Analysis for Bending Moments & SF’s
- Equivalent Frame - the Beams are the Slab width
- $K_{slab}$ = use full panel width for vertical loads.
- $K_{slab}$ = use 40% panel width for horizontal loads. Annex I.1.2.(1)

Load cases

NA – can use single load case provided:

- Variable load $\leq 1.25 \times$ Permanent load
- Variable load $\leq 5.0 \text{kN/m}^2$

Condition of using single load case is that Support BM’s should be reduced by 20% except at cantilever supports

Limitation of negative moments, $N_1$ and $N_2$
Analysis Methods

TR 64 - Figure 14
Reduction in maximum hogging moment at columns

Analysis Methods - Equi Frame

Distribution of Design Bending Moments, Annex I

<table>
<thead>
<tr>
<th>Table I.1</th>
<th>Column Strip</th>
<th>Middle Strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negative</td>
<td>60 - 80%</td>
<td>40 - 20%</td>
</tr>
<tr>
<td>Positive</td>
<td>50 - 70%</td>
<td>50 - 30%</td>
</tr>
</tbody>
</table>

$A_L = \text{Reinforcement area to resist full negative moment. Cl 9.4.1}$
Analysis Methods - Equi Frame

Distribution of Design Bending Moments - Example

Table I.1

<table>
<thead>
<tr>
<th>Column Strip</th>
<th>Middle Strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>75%</td>
<td>25%</td>
</tr>
</tbody>
</table>

$A_e = \text{Reinforcement area to resist full negative moment. Cl 9.4.1}$

$= 1600 \text{ mm}^2$

Column strip = 1200 mm$^2$  Middle strip = 400 mm$^2$

Equivalent frame method

Chosen solution: Flat Slab

Plan

Concept

Erection

Layout

Unit work time

Detail location

Cost survey

Member sizes

View plan

Flow sections

3D view

Project #: 1215  Handbook for D Valga

Master level 1 of 2

EC2 Webinar - Autumn 2016
Equivalent frame method

Analysis Methods

Equivalent frame method - Elastic Plane Frame

- Computer software normally used to assess bending moments and shear forces
- Design for full load in both directions
- RC spreadsheet TCC33.xls will carry out the analysis and design
Analysis Methods - Tabular Method

e.g. use coefficients from Concise Tables 15.2 to determine bending moments and shear forces. BM = coeff x n x span^2 \quad SF = coeff x n x span

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Location</th>
<th>End support/slabs connection</th>
<th>Internal supports and spans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Pinned end support</td>
<td>Continuous</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Outer support</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Near middle of end span</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>At 1st interior support</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>At middle of interior spans</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>At interior supports</td>
<td></td>
</tr>
<tr>
<td>Moment</td>
<td></td>
<td>0.0</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.086</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.086</td>
<td>0.063</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.063</td>
<td>0.063</td>
</tr>
<tr>
<td>Shear</td>
<td></td>
<td>0.40</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.046</td>
<td>0.063</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.063</td>
<td>0.50:0.50</td>
</tr>
</tbody>
</table>

- Design for full load in both directions
- Frame lateral stability must not be dependent on slab-col connections
- There must be at least three approx equal spans.
- Note: No column BM’s given in table.

---

**Concise Table 15.3**

- Suitable for regular grids and spans of similar length
- Design for full load in both directions
- Suitable for 2-span
- Note: No column BM’s given in table
Analysis Methods

Yield Line Method

Equilibrium and work methods.

‘work method’

External energy expended by the displacement of loads

\[ \text{External energy} = \text{Internal energy} \]

Internal energy dissipated by the yield lines rotating

Analysis Methods

Yield Line Method

Suitable for:

- irregular layouts
- Slabs supported on 2 or 3 edges only

Detailed guidance and numerous worked examples contained in:

*Practical Yield Line Design*

Deflection design to simplified rules
Analysis Methods

Finite Element Method

Suitable for:
- irregular layouts
- slabs with service openings
- post tensioned design (specialist software)

Common pitfalls:
- Use long term E-values (typically 1/3 to 1/2 short term value)
- Use cracked section properties (typically 1/2 gross properties) by adjusting E-value to suit
- Therefore appropriate E-values are usually 4 to 8 kN/mm²

Finite Element - Design moments

[Graph showing bending moment distribution]
Design Constraints

Punching Shear - EC2: cl 6.4 and cl 9.4.3

- Traditional links

- Shear Rails

Deflection:

Wherever possible use the span/effective depth ratios, cl 7.4.2 (2)

Span is based on the longer span and the K factor is 1.2

Reduction factor for brittle finishes for spans greater than 8.5m
Design Constraints

Moment Transfer from slab to column:

Edge and corner columns have limited capacity to transfer moments from slab - redistribution may be necessary (Annex I.1.2 (5), EC2 cl 9.4.2 & TR 64)

\[ M_{t, \text{max}} = 0.17 b_e d^2 f_{ck} \]

Effective width, \( b_e \).

Flat slab worked example

**Flexure**

From Worked Examples to EC2: Volume 1
Example 3.4.
Introduction to worked example

The slab is for an office where the specified load is 1.0 kN/m² for finishes and 4.0 kN/m² imposed (no partitions). Perimeter load is assumed to be 10 kN/m. Concrete is C30/37. The slab is 300 mm thick and columns are 400 mm square. The floor slabs are at 4.50 m vertical centres. A 2 hour fire rating is required.

This is example 3.4 of Worked examples to Eurocode 2: Volume 1.

Design information

- Nominal cover = 30mm

Design strip along grid line C

Determine the reinforcement - slab along grid line C.

Assume strip is 6 m wide
Slab is 300 mm deep
Flat slab
Worked example

For the previous flat slab example determine:

- Sagging reinforcement in the span 1-2

and

- Hoggling reinforcement at support 2

Analysis

Actions:

\[
g_k = 0.30 \times 25 + 1.0 = 8.5 \text{ kN/m}^2
\]

\[
q_k = 4.0 \text{ kN/m}^2
\]

\[
n = 1.25 \times 8.5 + 1.5 \times 4.0 = 16.6 \text{ kN/m}^2
\]

Analysis: using coefficients from Concise Table 15.3:

(Adjacent spans are 9.6 and 8.6 m. 8.6/9.6 = 0.89: i.e. > 85% so using coefficients is appropriate.)

Effective span = 9.6 - 2 x 0.4 + 2 x 0.3/2 = 9.5 m

In panel: sagging moment,

\[
M_{Ed} = (1.25 \times 8.5 \times 0.09 + 1.5 \times 4 \times 0.100) \times 6.0 \times 9.5^2 = 842.7 \text{ kNm}
\]

Along support 2: hoggling moment

\[
M_{Ed} = 16.6 \times 0.106 \times 6.0 \times 9.5^2 = 952.8 \text{ kNm}
\]

See Note to Concise Table 15.3 for support of 2-span slab
Division of moments

<table>
<thead>
<tr>
<th>Location</th>
<th>Negative moments</th>
<th>Positive moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column strip</td>
<td>60% – 80%</td>
<td>50% – 70%</td>
</tr>
<tr>
<td>Middle strip</td>
<td>40% – 20%</td>
<td>50% – 30%</td>
</tr>
</tbody>
</table>

Notes
The total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%.

The distribution of design moments given in BS 8110 (column strip: hogging 75%; sagging 55%; middle strip: hogging 25%, sagging 45%) may be used.

<table>
<thead>
<tr>
<th>MEd</th>
<th>Column strip</th>
<th>Middle strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ve sagging</td>
<td>0.50 x 842.7/3.0 = 140.5 kNm/m</td>
<td>0.50 x 842.7/3.0 = 140.5 kNm/m</td>
</tr>
</tbody>
</table>

(50% taken in both column and middle strips)

From analysis

3.4.5 Design grid line C

Effective depth, d:
\[ d = 300 - 30 - 20/2 = 260 \text{ mm} \]

a) Flexure: column strip and middle strip, sagging
\[ M_{Ed} = 140.5 \text{ kNm/m} \]
\[ K = M_{Ed}/bd^2f'\text{ck} = 140.5 \times 10^6/(1000 \times 260^2 \times 30) = 0.069 \]
\[ z/d = 0.94 \quad \text{(Using Concise table 15.5)} \]
\[ z = 0.94 \times 260 = 244 \text{ mm} \]
\[ A_e = M_{Ed}/f'_{ck}z = 140.5 \times 10^6/(244 \times 500/1.15) = 1324 \text{ mm}^2/\text{m} \]

\[ (\rho = 0.51\%) \]

Try H20 @ 200 B1 (1570 mm$^2$/m)

z = d [ 1 + (1 - 3.529K)$^{0.5}$]/2 = 260[1 + (1 - 3.529 \times 0.069)^{0.5}]/2 = 243 mm
Hogging Moments

<table>
<thead>
<tr>
<th></th>
<th>Column strip</th>
<th>Middle strip</th>
</tr>
</thead>
<tbody>
<tr>
<td>-ve hogging</td>
<td><strong>0.70 x 952.8/3.0</strong> = 222.3 kNm/m</td>
<td><strong>0.30 x 952.8/3.0</strong> = 95.3 kNm/m</td>
</tr>
</tbody>
</table>

(70% taken in column strip and 30% in middle strip)

c) **Flexure: column strip, hogging**

\[
M_{Ed} = 222.3 \text{ kNm/m}
\]

\[
K = \frac{M_{Ed}}{bd^2f_{ck}} = \frac{222.3 \times 10^6}{(1000 \times 260^2 \times 30)} = 0.109
\]

\[
z/d = 0.89 \quad \text{(Using Concise Table 15.5)}
\]

\[
z = 0.89 \times 260 = 231 \text{ mm}
\]

\[
A_s = \frac{M_{Ed}}{f_{yd}z} = \frac{222.3 \times 10^6}{(231 \times 500/1.15)} = 2213 \text{ mm}^2/m
\]

\[
z = d \left[ 1 + (1 - 3.529K)^{0.5} \right]/2 = 260 \left[ 1 + (1 - 3.529 \times 0.047)^{0.5} \right]/2 = 232 \text{ mm}
\]

d) **Flexure: middle strip, hogging**

\[
M_{Ed} = 95.3 \text{ kNm/m}
\]

\[
k = \frac{M_{Ed}}{bd^2f_{ck}} = \frac{95.3 \times 10^6}{(1000 \times 260^2 \times 30)} = 0.047
\]

\[
z/d = 0.95 \quad \text{(Using Concise table 15.5)}
\]

\[
z = 0.95 \times 260 = 247 \text{ mm}
\]

\[
A_s = \frac{M_{Ed}}{f_{yd}z} = \frac{95.3 \times 10^6}{(247 \times 500/1.15)} = 887 \text{ mm}^2/m
\]

\[
z = d \left[ 1 + (1 - 3.529K)^{0.5} \right]/2 = 260 \left[ 1 + (1 - 3.529 \times 0.047)^{0.5} \right]/2 = 248 \text{ mm} \leq 0.95d \leq 247 \text{ mm} \]

\[
z = 247
\]
Reinforcement distribution

Total area of reinforcement:
\[ A_{s,tot} = 2213 \times 3 + 887 \times 3 = 9300 \text{ mm}^2 \]
50% \[ A_{s,tot} = \frac{9300}{2} = 4650 \text{ mm}^2 \]
This is spread over a width of 1.5 m
\[ A_{s,req} = \frac{4650}{1.5} = 3100 \text{ mm}^2/m \]
Use H20 @ 100 ctrs T(3140 mm\(^2\)/m)

Remaining column strip:
\[ A_{s,req} = \frac{(2213 \times 3 - 4650)}{1.5} = 1326 \text{ mm}^2/m \]
Use H20 @ 200 ctrs T(1570 mm\(^2\)/m)
Or use H16 @ 100 ctrs(1540 mm\(^2\)/m)

Middle strip: \[ A_{s,req} = 887 \text{ mm}^2/m \]
Use H16 @ 200 ctrs T(1010 mm\(^2\)/m)
Or use H12 @ 100 ctrs (1130mm\(^2\)/m)

Exercise

Lecture 5

Check an edge column for punching shear
Punching shear Exercise

Based on the flat slab in section 3.4 of 
Worked Examples to EC2: Volume 1

Punching shear at column C1

400 mm Square Column
300 mm flat slab C30/37 concrete

Design information
- At C1 the ultimate column reaction is 609.5 kN
- Effective depths are 260mm & 240mm
- Reinforcement: \( \rho_y = 0.0080, \rho_z = 0.0069 \)
Punching shear exercise

For the previous flat slab example:

a) Check the shear stress at the perimeter of column C1. The $u_0$ perimeter.

b) Check the shear stress at the basic perimeter, $u_1$.

c) Determine the distance of the $u_{aux}$ perimeter from the face of column C1.

d) Determine the area of shear reinforcement required on a perimeter, i.e. find $A_{sw}$ for the $u_1$ perimeter.

Working space
End of Lecture 5