Slabs and Flat Slabs

Lecture 5

19th October 2016

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- **Designing for shear in slabs -** including punching shear
- **Detailing - Solid slabs**
- **Flat Slab Design -** includes flexure worked example
- **Exercise -** Punching shear

Designing for shear in slabs

- When shear reinforcement is not required – e.g. usually one and two-way spanning slabs
- Punching shear – e.g. flat slabs and pad foundations

Shear

There are three approaches to designing for shear:

- When shear reinforcement is not required e.g. usually slabs
- When shear reinforcement is required e.g. Beams, see Lecture 3
- Punching shear requirements e.g. flat slabs

The maximum shear strength in the UK should not exceed that of class C50/60 concrete

Shear resistance without shear reinforcement

Without Shear Reinforcement

EC2: Cl. 6.2.2 Concise: 7.2

$$V_{Rd,c} = [0.12k(100\,\rho_l f_{ck})^{1/3} + 0.15\sigma_{cp}]\,b_w d \qquad (6.2.a)$$

with a minimum of

$$V_{Rd,c} = (0.035k^{3/2} f_{ck}^{1/2} + 0.15\,\sigma_{cp})\,b_w d \qquad (6.2.b)$$

where:

k = $1 + \sqrt{(200/d)} \le 2.0$

ρ_l = $A_{sl}/b_w d \quad \le 0.02$

A_{sl} = area of the tensile reinforcement,

b_w = smallest width of the cross-section in the tensile area [mm]

σ_{cp} = $N_{Ed}/A_c < 0.2\,f_{cd}$ [MPa] Compression +ve

N_{Ed} = axial force in the cross-section due to loading or pre-stressing [in N]

A_c = area of concrete cross section [mm^2]

Shear - $v_{Rd,c}$

mpa The Concrete Centre

Concise Table 7.1 or 15.6

$v_{Rd,c}$ resistance of members without shear reinforcement, MPa											
$\frac{A_s}{(bd)}$ %	Effective depth, d (mm)										
	≤200	225	250	275	300	350	400	450	500	600	750
0.25	0.54	0.52	0.50	0.48	0.47	0.45	0.43	0.41	0.40	0.38	0.36
0.50	0.59	0.57	0.56	0.55	0.54	0.52	0.51	0.49	0.48	0.47	0.45
0.75	0.68	0.66	0.64	0.63	0.62	0.59	0.58	0.56	0.55	0.53	0.51
1.00	0.75	0.72	0.71	0.69	0.68	0.65	0.64	0.62	0.61	0.59	0.57
1.25	0.80	0.78	0.76	0.74	0.73	0.71	0.69	0.67	0.66	0.63	0.61
1.50	0.85	0.83	0.81	0.79	0.78	0.75	0.73	0.71	0.70	0.67	0.65
1.75	0.90	0.87	0.85	0.83	0.82	0.79	0.77	0.75	0.73	0.71	0.68
2.00	0.94	0.91	0.89	0.87	0.85	0.82	0.80	0.78	0.77	0.74	0.71
k	2.00	1.94	1.89	1.85	1.82	1.76	1.71	1.67	1.63	1.58	1.52
Table derived from: $v_{Rd,c} = 0.12\ k\ (100\rho_l f_{ck})^{(1/3)} \geq 0.035\ k^{1.5}\ f_{ck}^{0.5}$ where $k = 1 + \sqrt{(200/d)} \leq 2$ and $\rho_l = A_s/(bd) \leq 0.02$ Note: This table has been prepared for $f_{ck} = 30$. Where ρ_l exceeds 0.40% the following factors may be used:											
f_{ck}	25	28	32	35	40	45	50				
factor	0.94	0.98	1.02	1.05	1.10	1.14	1.19				

Shear in Slabs

mpa The Concrete Centre

Most slabs do not require shear reinforcement

$\therefore$ Check $V_{Ed} < V_{Rd,c}$

Where $V_{Rd,c}$ is shear resistance of members without reinforcement

$$v_{Rd,c} = 0.12\ k\ (100\ \rho_l\ f_{ck})^{1/3} \geq 0.035\ k^{1.5}\ f_{ck}^{0.5}$$

Where $V_{Ed} > V_{Rd,c}$, shear reinforcement is required and the strut inclination method should be used

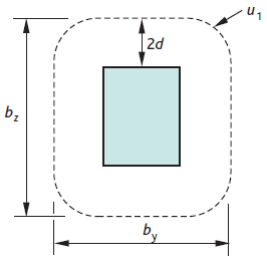
$\rho_l = A_s/(bd)$	Effective depth, d (mm)										
	≤200	225	250	275	300	350	400	450	500	600	750
0.25%	0.54	0.52	0.50	0.48	0.47	0.45	0.43	0.41	0.40	0.38	0.36
0.50%	0.59	0.57	0.56	0.55	0.54	0.52	0.51	0.49	0.48	0.47	0.45
0.75%	0.68	0.66	0.64	0.63	0.62	0.59	0.58	0.56	0.55	0.53	0.51
1.00%	0.75	0.72	0.71	0.69	0.68	0.65	0.64	0.62	0.61	0.59	0.57
1.25%	0.80	0.78	0.76	0.74	0.73	0.71	0.69	0.67	0.66	0.63	0.61
1.50%	0.85	0.83	0.81	0.79	0.78	0.75	0.73	0.71	0.70	0.67	0.65
1.75%	0.90	0.87	0.85	0.83	0.82	0.79	0.77	0.75	0.73	0.71	0.68
2.00%	0.94	0.91	0.89	0.87	0.85	0.82	0.80	0.78	0.77	0.74	0.71
2.50%	0.94	0.91	0.89	0.87	0.85	0.82	0.80	0.78	0.77	0.74	0.71

How-to Compendium p21

Punching shear

Punching shear symbols

u_i = i^{th} perimeter
u_1 = basic control perimeter at 2d
u_{1^*} = reduced basic control perimeter
u_0 = column perimeter
d = average effective depth
k = coeff. depending on column shape –see Table 6.1
W_1 = a shear distribution factor – see 6.4.3(3)



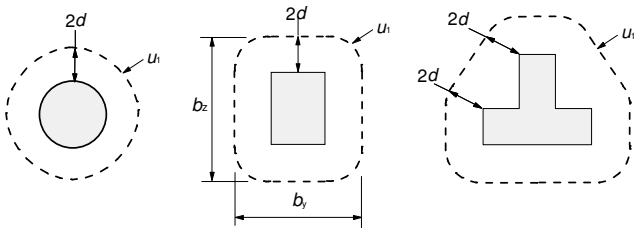
Punching Shear

EC2:Cl. 6.4

Concise: Figure 8.3

Punching shear does not use the *Variable Strut inclination method* and is similar to BS 8110 methods

- The basic control perimeter is set at 2*d* from the loaded area
- The shape of control perimeters have rounded corners



- Where shear reinforcement is required the shear resistance is the sum of the concrete and shear reinforcement resistances.

Punching Shear

EC2: Cl. 6.4.3 & 6.4.4

6.4.3 (2)

(b) Punching shear reinforcement is not necessary if:

$V_{Ed} \leq V_{Rd,c}$

When calculating $v_{Rd,c}$:

6.4.4 (1)

$$\rho_l = \sqrt{\rho_{ly} \cdot \rho_{lz}} \leq 0{,}02$$

ρ_{ly}, ρ_{lz} relate to the bonded tension steel in y- and z- directions respectively. The values ρ_{ly} and ρ_{lz} should be calculated as mean values taking into account a slab width equal to the column width plus 3d each side.

Punching Shear

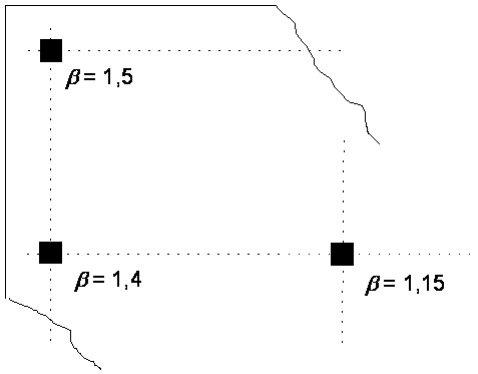
The applied shear stress should be taken as:

$v_{Ed} = \beta V_{Ed}/u_i d$ where $\beta = 1 + k(M_{Ed}/V_{Ed})u_1/W_1$

For structures where:

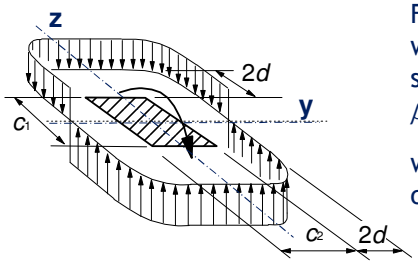
- lateral stability does not depend on frame action
- adjacent spans do not differ by more than 25%

the approximate values for β shown may be used:



Punching Shear

Where the simplified arrangement is not applicable then β can be calculated



For a rectangular internal column with biaxial bending the following simplification may be used:

$\beta = 1 + 1.8\{(e_y/b_z)^2 + (e_z/b_y)^2\}^{0.5}$

where b_y and b_z are the dimensions of the control perimeter

For other situations there is plenty of guidance on determining β given in Cl 6.4.3 of the Code.

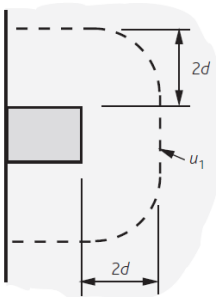
Punching shear control perimeters

Basic perimeter, u_1

EC2: Cl. 6.4.2

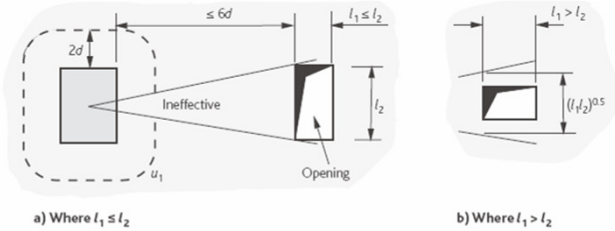
Near to an edge

Concise: Figure 8.4



Near to an opening

Concise: Figure 8.6

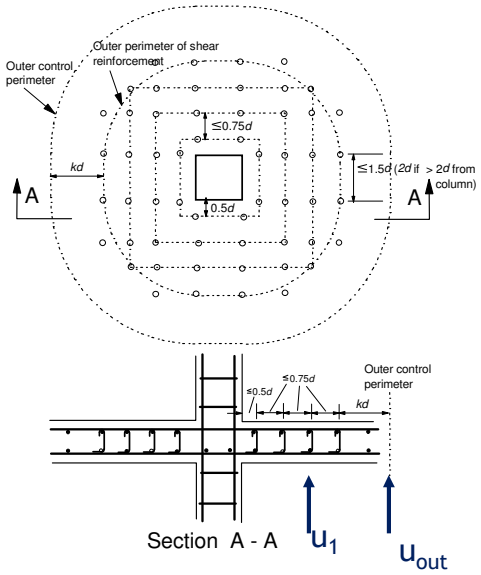


a) Where $l_1 \le l_2$ b) Where $l_1 > l_2$

Punching Shear Reinforcement

EC2: Cl. 6.4.5

Concise: Figures 12.5 & 12.6



The outer control perimeter at which shear reinforcement is not required, should be calculated from:

$$u_{out,ef} = \beta V_{Ed} / (v_{Rd,c}\, d)$$

The outermost perimeter of shear reinforcement should be placed at a distance not greater than kd (k = 1.5) within the outer control perimeter.

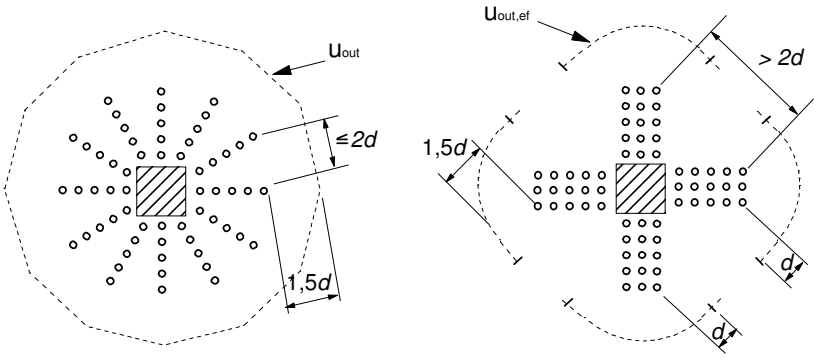
Punching Shear Reinforcement

EC2: Cl. 6.4.5

Concise; Figure 8.10

Where proprietary systems are used the control perimeter at which shear reinforcement is not required, u_{out} or $u_{out,ef}$ (see Figure) should be calculated from the following expression:

$$u_{out,ef} = \beta V_{Ed} / (v_{Rd,c}\, d)$$



Punching Shear Reinforcement

EC 2: Cl. 6.4.5, Equ 6.52 Concise: 8.5

Where shear reinforcement is required it should be calculated in accordance with the following expression:

$v_{Rd,cs} = 0.75\, v_{Rd,c} + 1.5\,(d/s_r)\, A_{sw} f_{ywd,ef}\,(1/(u_1 d))\,\sin\alpha$ (6.52)

A_{sw} = area of shear reinforcement in each perimeter around the col.

s_r = radial spacing of layers of shear reinforcement

α = angle between the shear reinforcement and the plane of slab

$f_{ywd,ef}$ = effective design strength of the punching shear reinforcement,
$= 250 + 0.25\, d \le f_{ywd}$ (MPa.)

d = mean effective depth of the slabs (mm)

Max. shear stress at column face, $v_{Ed} = \dfrac{\beta V_{Ed}}{u_0 d} \le v_{Rd,max} = 0.5\, \nu f_{cd}$

EC2 Equ 6.53

Punching Shear Reinforcement

EC 2: Cl. 6.4.5 (3), Equ 6.53 Concise: 8.6

Max. shear stress at column face, the u_0 perimeter $v_{Ed} = \dfrac{\beta V_{Ed}}{u_0 d} \le v_{Rd,max} = 0.5\, \nu f_{cd}$

where

β = factor dealing with eccentricity (see

V_{Ed} = applied shear force

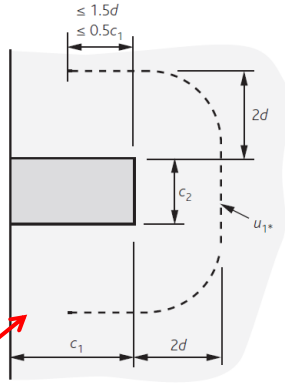
d = mean effective depth

u_0 = $2(c_1 + c_2)$ for interior columns
= $c_2 + 3d \le c_2 + 2c_1$ for edge columns
= $3d \le c_2 + 2c_1$ for corner columns

where

c_1 = column depth

c_2 = column width



a) Edge column

c_1 and c_2 are illustrated in Concise Figure 8.5

Punching Shear Reinforcement

Check $v_{Ed} \leq 2\ v_{Rdc}$ at basic control perimeter (NA check)

Note: UK NA says 'first' control perimeter, but the paper on which this guidance is based says 'basic' control perimeter*

The minimum area of a link leg (or equivalent), $A_{sw,min}$, is given by the following expression:

$A_{sw,min}\ (1.5 \sin\alpha + \cos\alpha)/(s_r\ s_t) \geq (0{,}08\ \sqrt{(f_{ck})})/f_{yk}$ EC2 equ 9.11

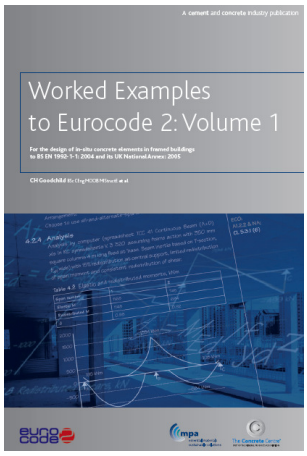
$A_{sw,min} \geq (0{,}053\ s_r\ s_t\ \sqrt{(f_{ck})})\ /f_{yk}$ For vertical links

***FRASER, AS & JONES, AEK. Effectiveness of punching shear reinforcement to EN 1992-1-1:2004. The Structural Engineer ,19 May 2009.**

Punching shear Worked example

From Worked Examples to EC2: Volume 1

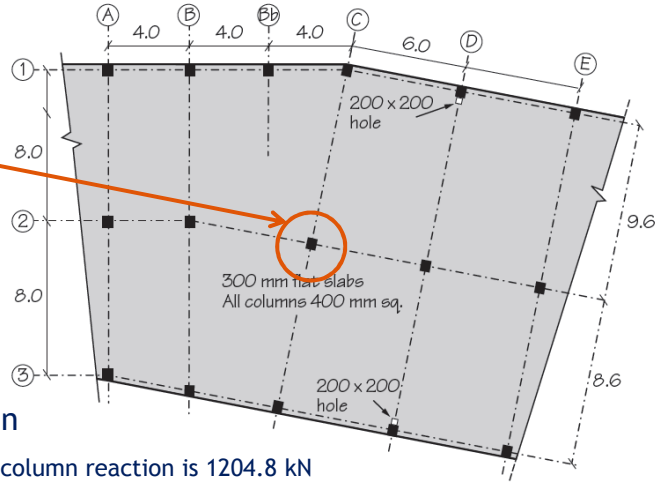
Example 3.4.10



Punching shear at column C2

400 mm Square Column

300 mm flat slab C30/37 concrete

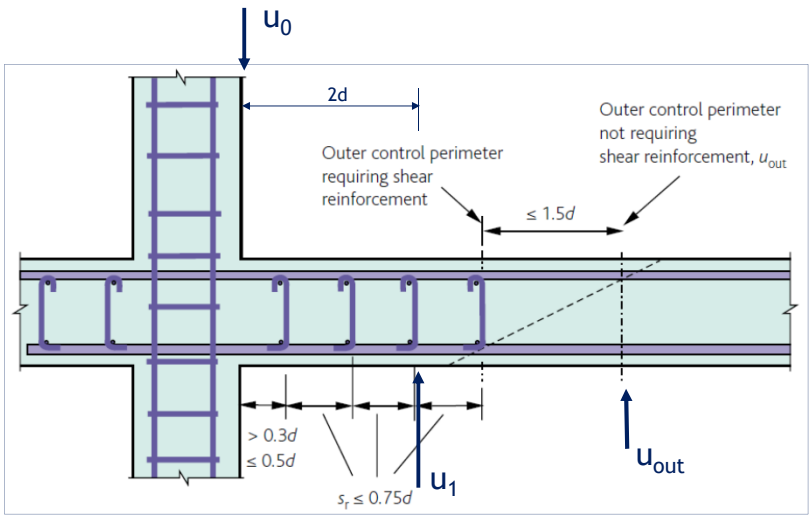


Design information

- At C2 the ultimate column reaction is 1204.8 kN
- Effective depths are 260mm & 240mm
- Reinforcement: ρ_{ly} = 0.0085, ρ_{lz} = 0.0048

Punching shear

A few definitions:



Solution

1. Check shear at the perimeter of the column

v_{Ed} = $\beta V_{Ed}/(u_0 d) < v_{Rd,max}$

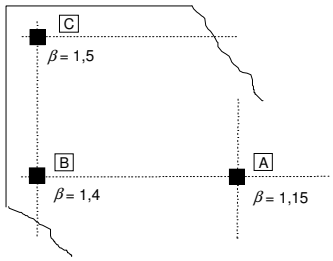
β = 1.15

u_0 = 4 x 400 = 1600 mm

d = (260 + 240)/2 = 250 mm

v_{Ed} = 1.15 x 1204.8 x 1000/(1600 x 250)

= 3.46 MPa



$v_{Rd,max}$ = 0.5 νf_{cd}

= 0.5 x 0.6(1-f_{ck}/250) x $\alpha_{cc} f_{ck}/\gamma_m$

= 0.5 x 0.6(1-30/250) x 1.0 x 30 /1.5 = 5.28 MPa

v_{Ed} < $v_{Rd,max}$ ...OK

Solution

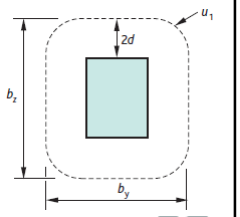
2. Check shear at u_1, the basic control perimeter

v_{Ed} = $\beta V_{Ed}/(u_1 d) < v_{Rd,c}$

β, V_{Ed} as before

u_1 = 2(c_x + c_y) + 2π x 2d

= 2(400 + 400) + 2π x 2 x 250 = 4742 mm



v_{Ed} = 1.15 x 1204.8 x 1000/(4742 x 250) = 1.17 MPa

$v_{Rd,c}$ = 0.12 $k(100\rho_l f_{ck})^{1/3}$

k = 1 + $(200/d)^{1/2}$ = 1 + $(200/250)^{1/2}$ = 1.89

ρ_l = $(\rho_{ly}\rho_{lx})^{1/2}$ = (0.0085 x 0.0048) $^{1/2}$ = 0.0064

ρ from flexural calcs

$v_{Rd,c}$ = 0.12 x 1.89(100 x 0.0064 x 30)$^{1/3}$ = 0.61 MPa

v_{Ed} > $v_{Rd,c}$?

1.17 MPa > 0.61 MPa ... Therefore punching shear reinf. required

2a. NA check:

v_{Ed} ≤ 2$v_{Rd,c}$ at basic control perimeter

1.17 MPa ≤ 2 x 0.61 MPa = 1.22 MPa - OK

Solution

3. Perimeter at which punching shear no longer required

u_{out} = $\beta V_{Ed}/(dv_{Rd,c})$

= 1.15 x 1204.8 x 1000/(250 x 0.61)

= 9085 mm

Rearrange: $u_{out} = 2(c_x + c_y) + 2\pi r_{out}$

r_{out} = $(u_{out} - 2(c_x + c_y))/(2\pi)$

= (9085 – 1600)/(2π) = 1191 mm

Position of outer perimeter of reinforcement from column face:

r = 1191 – 1.5 x 250 = 816 mm

Maximum radial spacing of reinforcement:

$s_{r,max}$ = 0.75 x 250 = 187 mm, say 175 mm

Solution

4. Area of reinforcement

$A_{sw} \geq (v_{Ed} - 0.75v_{Rd,c})s_r u_1/(1.5f_{ywd,ef})$

$f_{ywd,ef}$ = $(250 + 0.25d)$ = 312 MPa

$A_{sw} \geq (1.17 - 0.75 \times 0.61) \times 175 \times 4741/(1.5 \times 312)$

≥ 1263 mm²/perim.

1H10 is 78.5 mm² dia.

16H10 = 1256 mm² / per perim

See layout on next slide

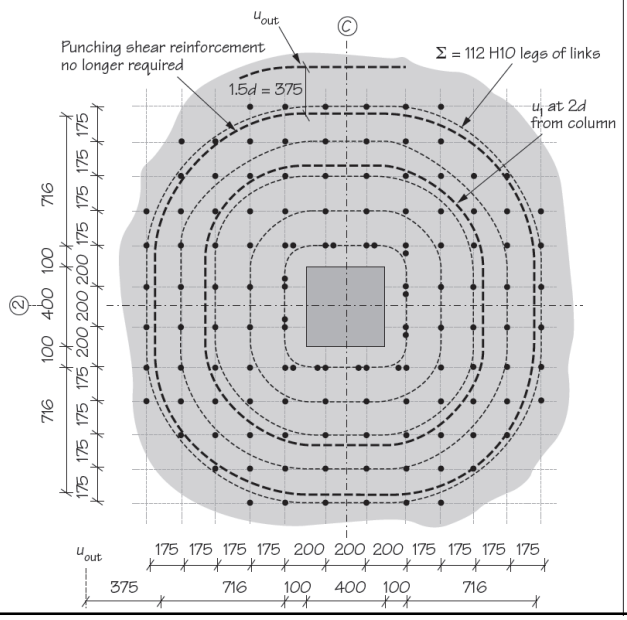
Minimum area of a link leg:

$A_{sw,min} \geq (0.053\ s_r\ s_t\ \sqrt{(f_{ck})})\ /f_{yk}$ = 0.053 x 175 x 350 x √30 / 500

≥ 36 mm²

could use H8s (50 mm²) but would need 26 per perimeter So use H10s (same price as H8s!)

Solution



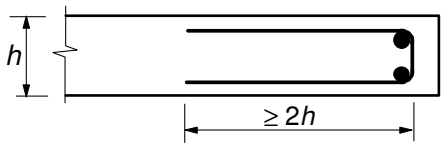
Detailing - Solid slabs

Detailing – Solid slabs

EC2: Cl. 9.3

Rules for one-way and two-way solid slabs

- Generally: as for beams.
- Where partial fixity exists, but not taken into account in design:
  Internal supports: $A_{s,top} \geq 0{,}25A_s$ for M_{max} in adjacent span
  End supports: $A_{s,top} \geq 0{,}15A_s$ for M_{max} in adjacent span
- This top reinforcement should extend ≥ 0,2 adjacent span
- Reinforcement at free edges should include 'u' bars and longitudinal bars



- Secondary reinforcement 20%

Flat Slab Design

Flat Slab Design - Contents

Flat slabs - Introduction

EC2 particular rules for flat slabs

Initial sizing

Analysis methods - BM's and Shear Force

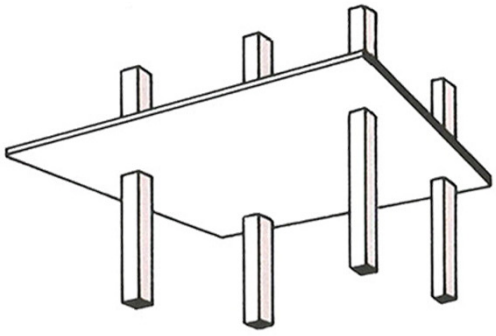
Design constraints

- Punching shear
- Deflection
- Moment transfer from slab to column

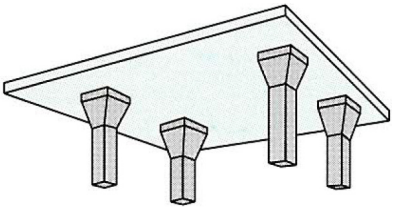
Flat Slabs - Introduction

What are flat slabs?

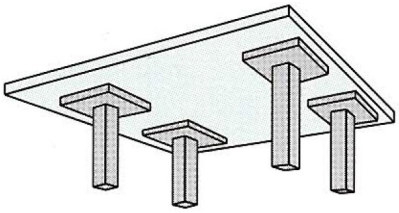
- Solid concrete floors of constant thickness
- They have flat soffits



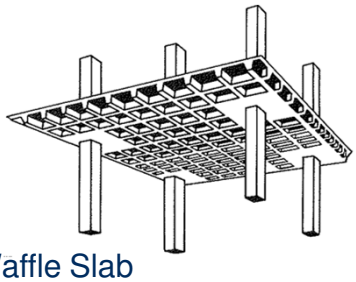
Flat Slabs - Introduction



Column Head



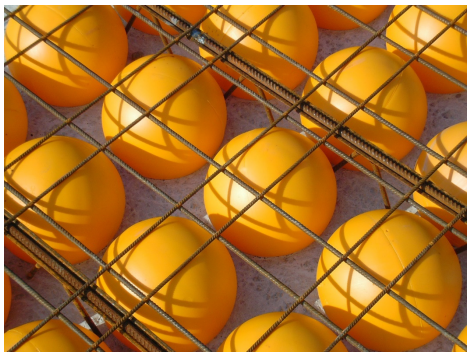
Drop Panel



Waffle Slab

Flat Slabs - Introduction

VOIDED SLABS



1. COBIAX
2. BUBBLEDECK

Poll Q1:
Shear resistance of a beam section

What is the shear resistance governed by the crushing of compression struts?

a. V_{Ed}

b. $V_{Rd,c}$

c. $V_{Rd,max}$

d. $V_{Rd,s}$

Particular rules for flat slabs

EC2 sections relevant to Flat Slabs:

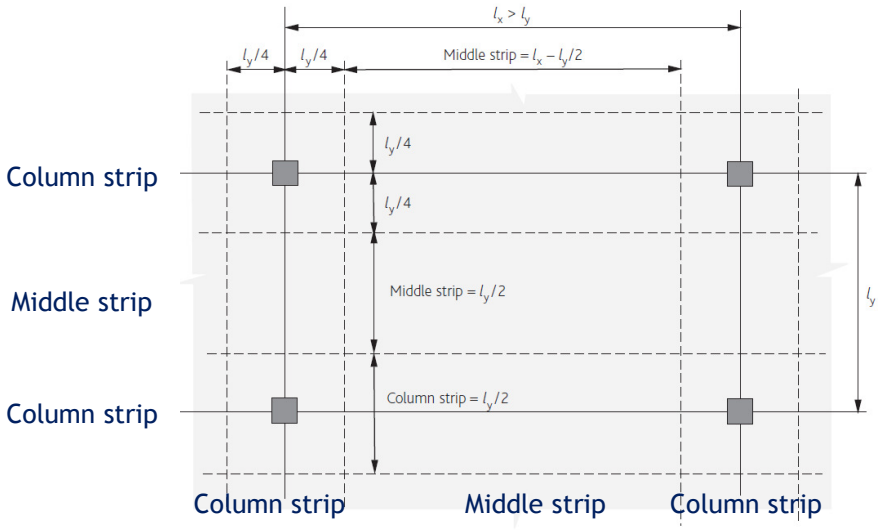
- **Section 6 Ultimate Limit States**
  - cl 6.4 Punching (shear) & PD 6687 cl 2.16, 2.17 & 2.1.8
- **Section 9 Detailing of members and particular rules**
  - Cl 9.4 Flat slabs
    - 9.4.1 Slab at internal columns
    - 9.4.2 Slab at edge and corner columns
    - 9.4.3 Punching shear reinforcement
- **Annex I (Informative) Analysis of flat slabs and shear walls**
  - **I.1 Flat Slabs**
    - I.1.1 General
    - I.1.2 Equivalent frame analysis
    - I.1.3 Irregular column layout

The Concrete Society, Technical Report 64 - Guide to the Design and Construction of Reinforced Concrete Flat Slabs

Particular rules for flat slabs
Distribution of moments

EC2: Figure I.1

Concise Figure 5.11



Particular rules for flat slabs
Distribution of moments

EC2: Table I.1

Concise: Table 5.2

Location	Negative moments	Positive moments
Column strip	60% – 80%	50% – 70%
Middle strip	40% – 20%	50% – 30%

Notes

The total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%

The distribution of design moments given in BS 8110 (column strip: hogging 75%, sagging 55%; middle strip: hogging 25%, sagging 45%) may be used

Particular rules for flat slabs

EC2: Cl. 9.4

Concise: 12.4.1

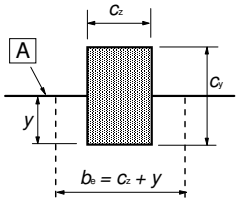
- Arrangement of reinforcement should reflect behaviour under working conditions.
- At internal columns $0.5A_t$ should be placed in a width = $0.25 \times$ panel width.
- At least two bottom bars should pass through internal columns in each orthogonal directions.

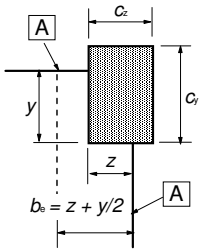
Particular rules for flat slabs

EC2: Figure 9.9, I.1.2(5)

Concise Figure 5.12

- Design reinforcement at edge and corner reinforcement should be placed within b_e





- The maximum moment that can be transferred from the slab to the column should be limited to $0.17 b_e d^2 f_{ck}$

Moment transfer

Edge and corner columns have limited capacity to transfer moments from slab – redistribution may be necessary

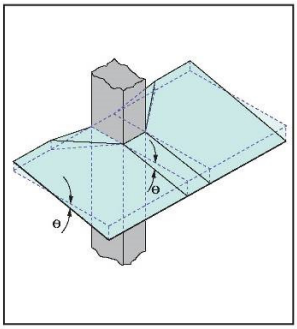
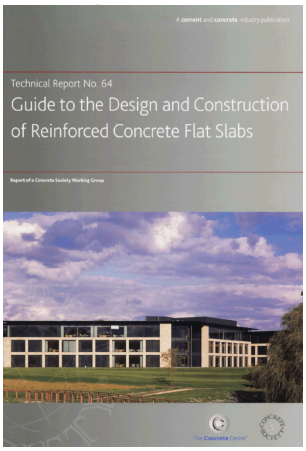
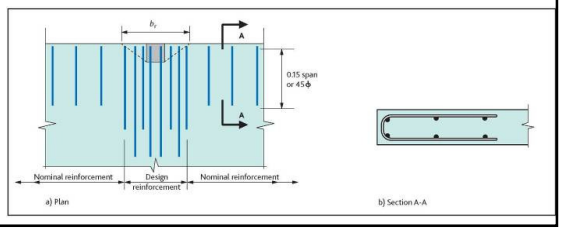


Figure 8





Rebar arrangement
Figure 47

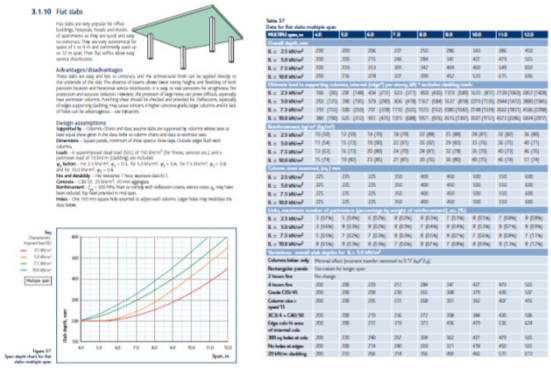
Initial sizing

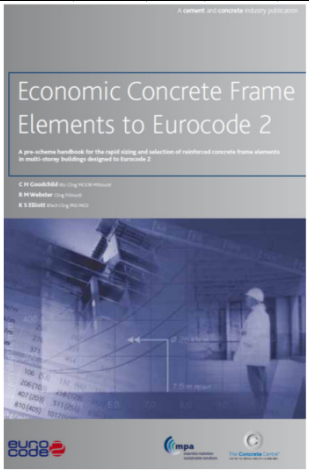
mpa The Concrete Centre

Imposed Load, Q_k (kN/m^2)	2.5	5	7.5	10
Multiple Span	28	26	25	23

3 methods:

1. Simple span to depth table
2. Use *Economic Concrete Frame Elements*

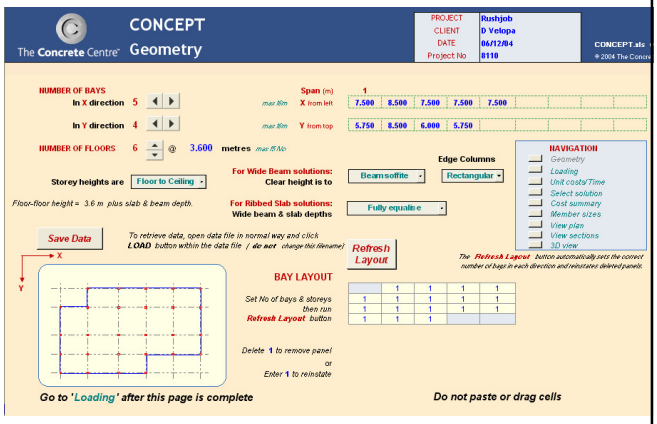




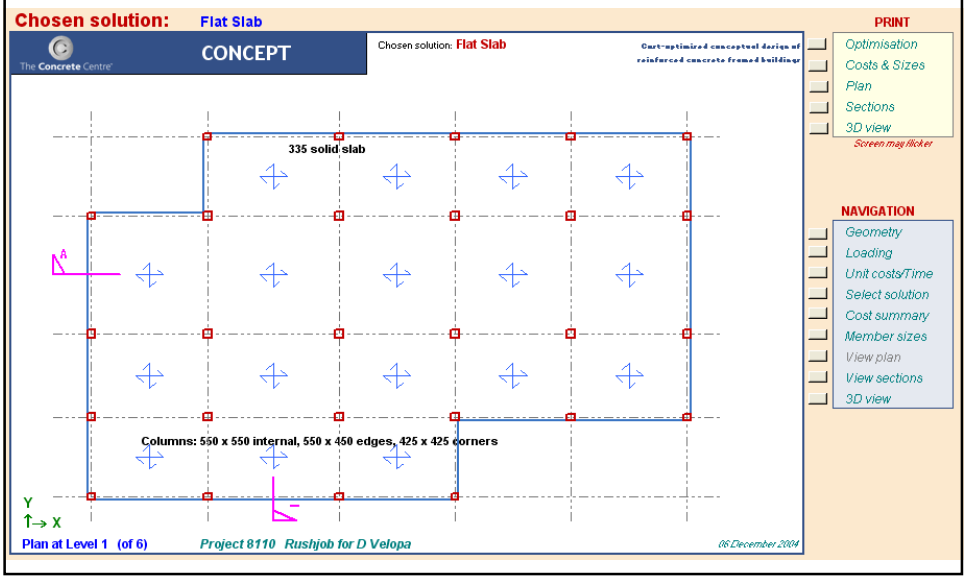
Initial sizing

3 methods:

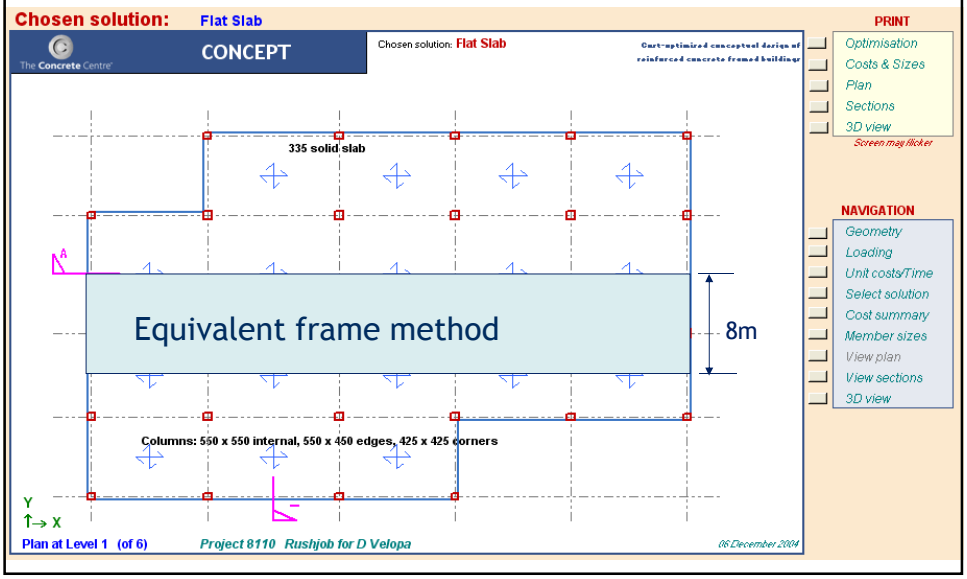
1. Simple span to depth table
2. Use *Economic Concrete Frame Elements*
3. Use *Concept.xls*



Initial sizing



Initial sizing



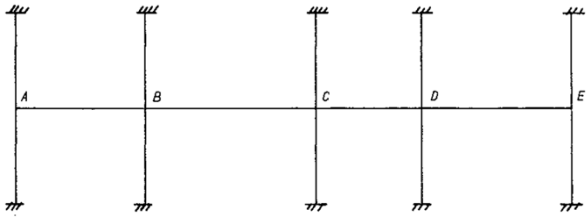
Analysis Methods

- Elastic Plane Frame – Equivalent Frame Method, Annex I
- Tabular Method - Equivalent Frame Method, Annex I
- Yield Line
  - Plastic method of design
- Finite Element Analysis
  - Elastic method
  - Elasto plastic

Analysis Methods

Elastic Plane Frame – **Equivalent Frame Method, Annex I**

- Apply in both directions – Y and Z
- Method of Analysis for Bending Moments & SF's
- Equivalent Frame - the Beams are the Slab width
- K_{slab} = use full panel width for vertical loads.
- K_{slab} = use 40% panel width for horizontal loads. Annex I.1.2.(1)



Analysis Methods

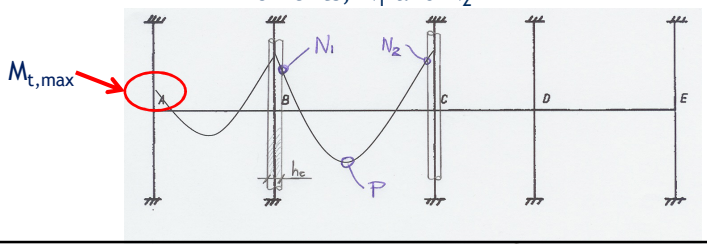
Load cases

NA – can use single load case provided:

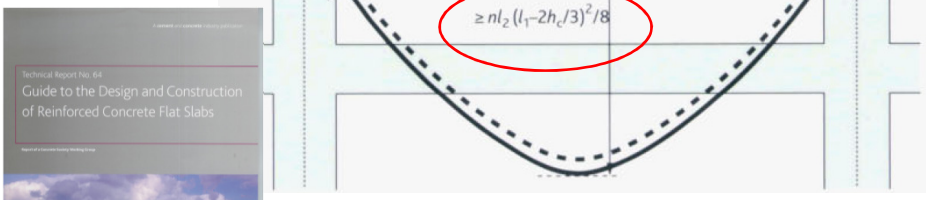
- Variable load ≤ 1.25 x Permanent load
- Variable load ≤ 5.0 kN/m2

Condition of using single load case is that Support BM's should be reduced by 20% except at cantilever supports

Limitation of negative moments, N_1 and N_2



Analysis Methods



TR 64 – Figure 14

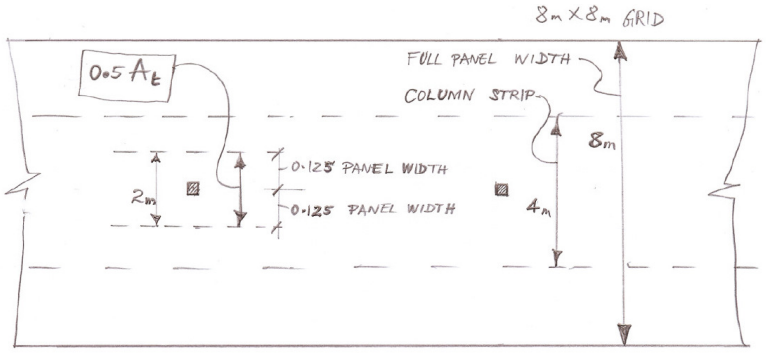
Reduction in maximum hogging moment at columns

Analysis Methods - Equi Frame

Distribution of Design Bending Moments, Annex I

Table I.1	Column Strip	Middle Strip
Negative	60 - 80%	40 - 20%
Positive	50 - 70%	50 - 30%

A_t = Reinforcement area to resist full negative moment. Cl 9.4.1

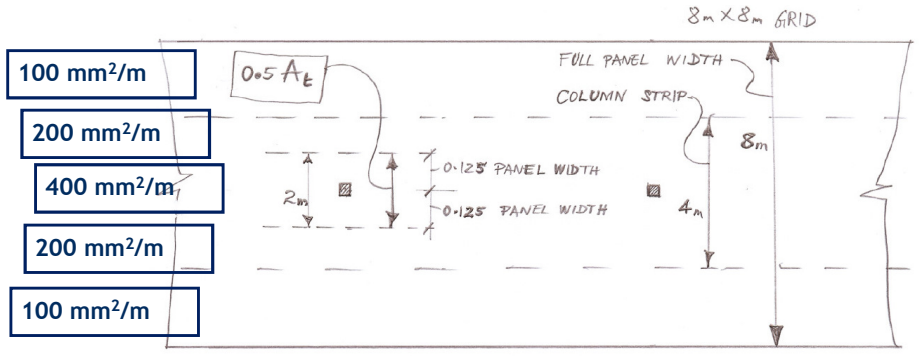


Analysis Methods – Equi Frame

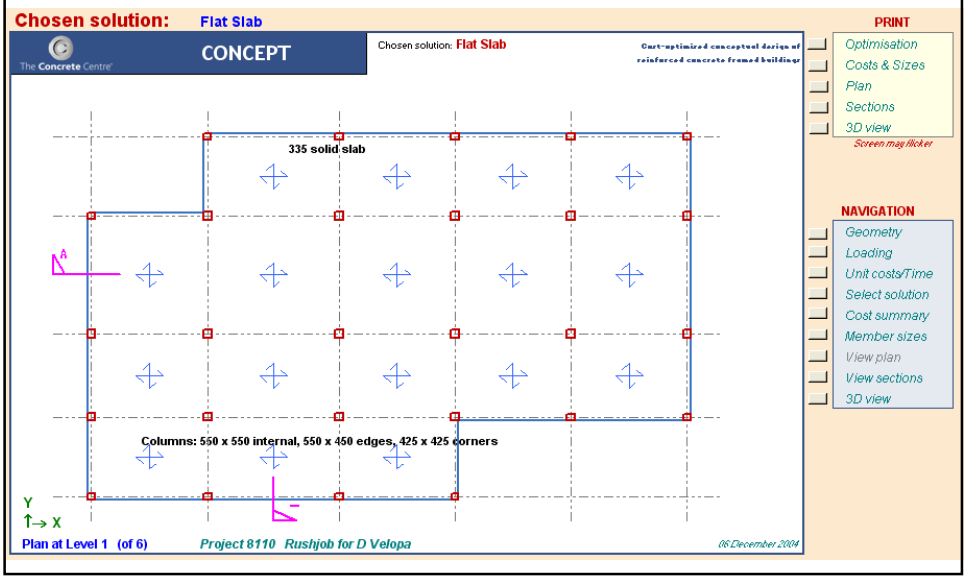
Distribution of Design Bending Moments - Example

Table I.1	Column Strip	Middle Strip
Negative	75%	25%

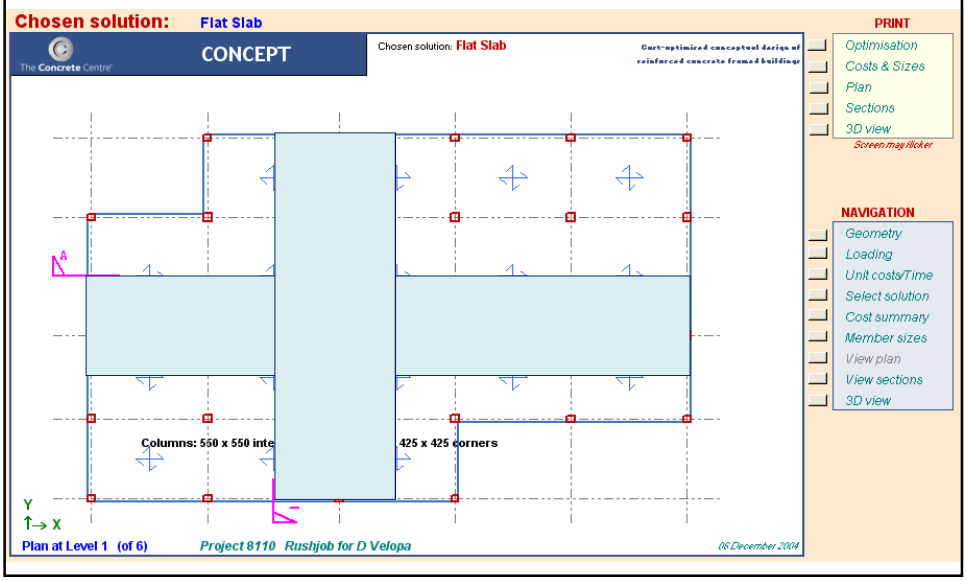
A_t = Reinforcement area to resist full negative moment. Cl 9.4.1
= 1600 mm²
Column strip = 1200 mm² Middle strip = 400 mm²



Equivalent frame method



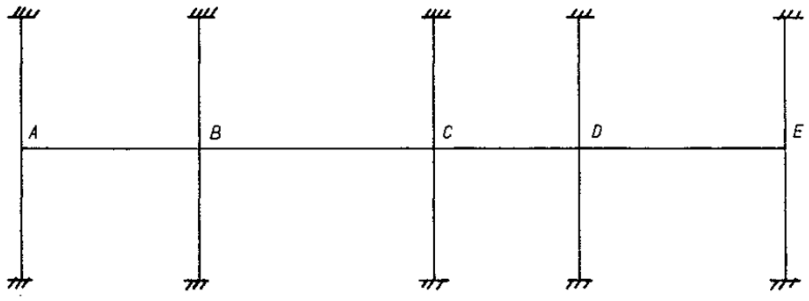
Equivalent frame method



Analysis Methods

Equivalent frame method - Elastic Plane Frame

- Computer software normally used to assess bending moments and shear forces
- Design for *full* load in *both* directions
- RC spreadsheet TCC33.xls will carry out the analysis and design



Analysis Methods - Tabular Method

e.g. use coefficients from Concise Tables 15.2 to determine bending moments and shear forces. BM = coeff *x* n *x* span^2 SF = coeff *x* n *x* span

Table 15.2
Coefficients for use with one-way spanning slabs to Eurocode 2

Coefficient	Location						
	End support/slab connection				Internal supports and spans		
	Pinned end support		Continuous				
	Outer support	Near middle of end span	Outer support	Near middle of end span	At 1st interior support	At middle of interior spans	At interior supports
Moment	0.0	0.086	– 0.04	0.075	– 0.086	0.063	– 0.063
Shear	0.40	—	0.46	—	0.60:0.60	—	0.50:0.50

- Design for *full* load in *both* directions
- Frame lateral stability must not be dependent on slab-col connections
- There must be at least three approx equal spans.
- Note: No column BM's given in table.

Analysis Methods - Tabular Method

A little more accurate:

Table 15.3
Coefficients for use with beams (and one-way spanning slabs) to Eurocode 2

Coefficient	Location				
	Outer support	Near middle of end span	At 1st interior support	At middle of interior spans	At interior supports
Moment g_k and q_k	25% span[a]	—	0.094	—	0.075
Moment g_k	—	0.090	—	0.066	—
Moment q_k	—	0.100	—	0.086	—
Shear	0.45	—	0.63:0.55	—	0.50:0.50[b]

Conditions
For beams and slabs, 3 or more spans. (They may also be used for 2 span beams but support moment coefficient = 0.106 and internal shear coefficient = 0.63 both sides).

Generally $Q_k \leq G_k$, and the loading should be substantially uniformly distributed. Otherwise special curtailment of reinforcement is required.

Minimum span ≥ 0.85 x maximum (and design) span.

Design moment at supports = coeff x *n* x span^2 or
in spans = (coeff g_k x $\gamma_G g_k$ + coeff q_k x $\psi/\gamma_Q q_k$) x span^2.

Design shear at centreline of supports = coeff x *n* x span where *n* is a UDL with a single variable action = $\gamma_G g_k + \psi/\gamma_Q q_k$ where g_k and q_k are characteristic permanent and variable actions in kN/m.

γ_G and ψ/γ_Q are dependent on use of BS EN 1990 Exp. (6.10), Exp. (6.10a) or Exp. (6.10b). See Section 15.1.

Basis: All- and alternate-spans-loaded cases as UK National Annex and 15% redistribution at supports.

Key
a At outer support '25% span' relates to the UK Nationally Determined Parameter for BS EN 1992-1-1 9.2.1.2(1) for minimum percentage of span bending moment to be assumed at supports in beams in monolithic construction. 15% may be appropriate for slabs (see BS EN 1992-1-1 Cl 9.3.1.2)

Concise Table 15.3

– Suitable for regular grids and spans of similar length

– Design for *full* load in *both* directions

– Suitable for 2-span

– Note: No column BM's given in table

Analysis Methods

Yield Line Method

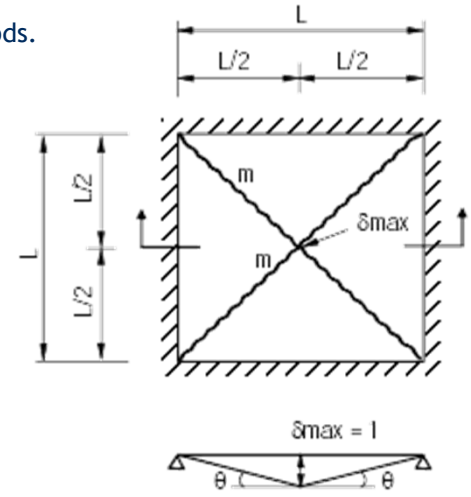
Equilibrium and work methods.

'work method'

External energy expended by the displacement of loads

=

Internal energy dissipated by the yield lines rotating



Analysis Methods

Yield Line Method

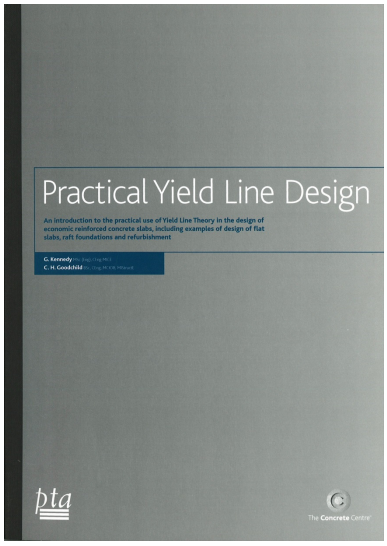
Suitable for:

- irregular layouts
- Slabs supported on 2 or 3 edges only

Detailed guidance and numerous worked examples contained in:

Practical Yield Line Design

Deflection design to simplified rules

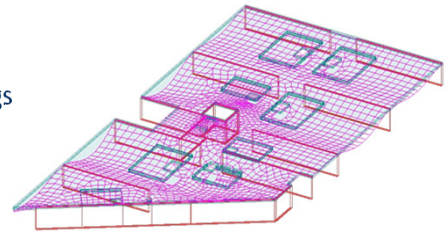


Analysis Methods

Finite Element Method

Suitable for:

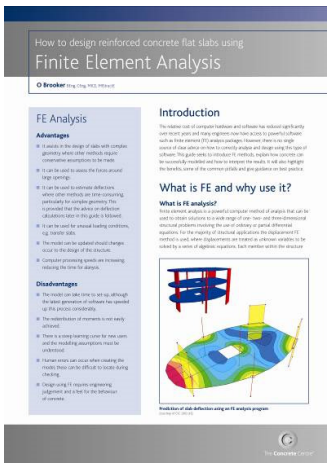
- irregular layouts
- slabs with service openings
- post tensioned design (specialist software)

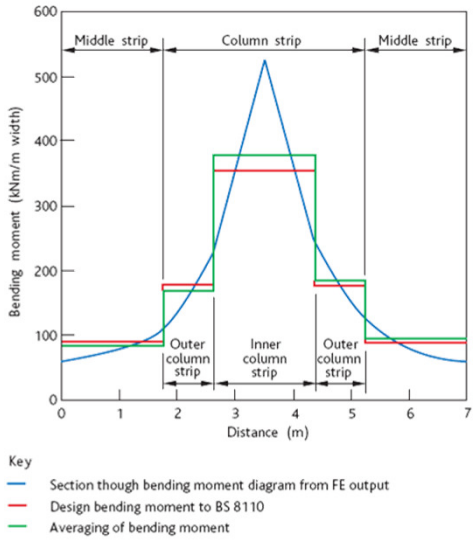


Common pitfalls:

- Use long term E-values (typically 1/3 to 1/2 short term value)
- Use cracked section properties (typically 1/2 gross properties) by adjusting E-value to suit
- Therefore appropriate E-values are usually 4 to 8 kN/mm^2

Finite Element - Design moments





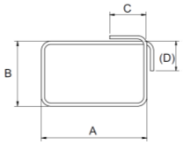
Key

- Section though bending moment diagram from FE output
- Design bending moment to BS 8110
- Averaging of bending moment

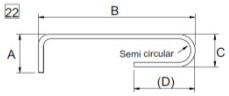
Design Constraints

Punching Shear - EC2: cl 6.4 and cl 9.4.3

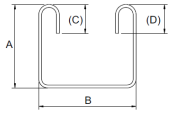
- Traditional links



Shape 51

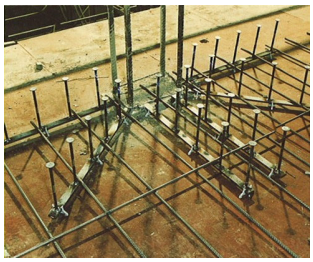


Shape 22



Shape 47

- Shear Rails



Design Constraints

Deflection:

Wherever possible use the span/effective depth ratios, cl 7.4.2 (2)

Span is based on the longer span and the K factor is 1.2

Reduction factor for brittle finishes for spans greater than 8.5m

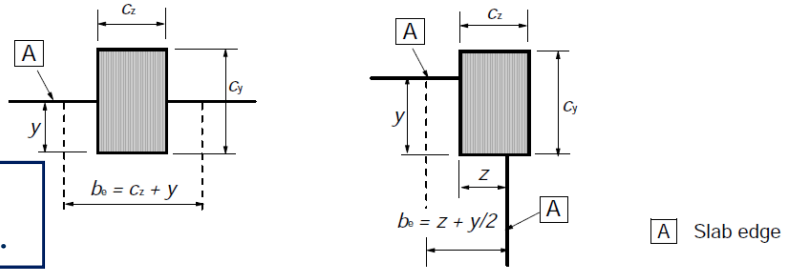
Design Constraints

Moment Transfer from slab to column:

Edge and corner columns have limited capacity to transfer moments from slab – redistribution may be necessary (Annex I.1.2 (5), EC2 cl 9.4.2 & TR 64)

$M_{t,\,max} = 0.17\, b_e\, d^2\, f_{ck}$

Effective width, b_e.



Note: y can be > c_y

a) Edge column

Note: z can be > c_z and y can be > c_y

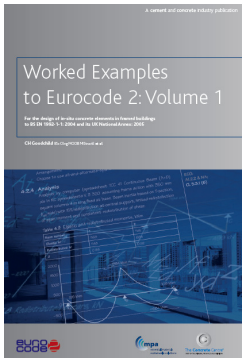
b) Corner column

Flat slab worked example

Flexure

From Worked Examples to EC2: Volume 1

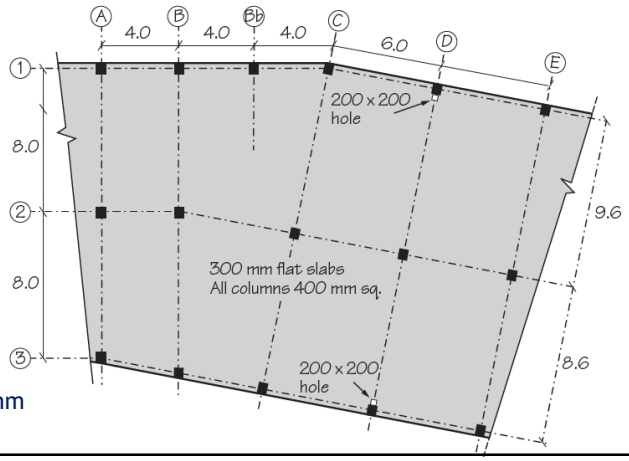
Example 3.4.



Introduction to worked example

The slab is for an office where the specified load is 1.0 kN/m^2 for finishes and 4.0 kN/m^2 imposed (no partitions). Perimeter load is assumed to be 10 kN/m. Concrete is C30/37. The slab is 300 mm thick and columns are 400 mm square. The floor slabs are at 4.50 m vertical centres. A 2 hour fire rating is required.

This is example 3.4 of *Worked examples to Eurocode 2: Volume 1.*

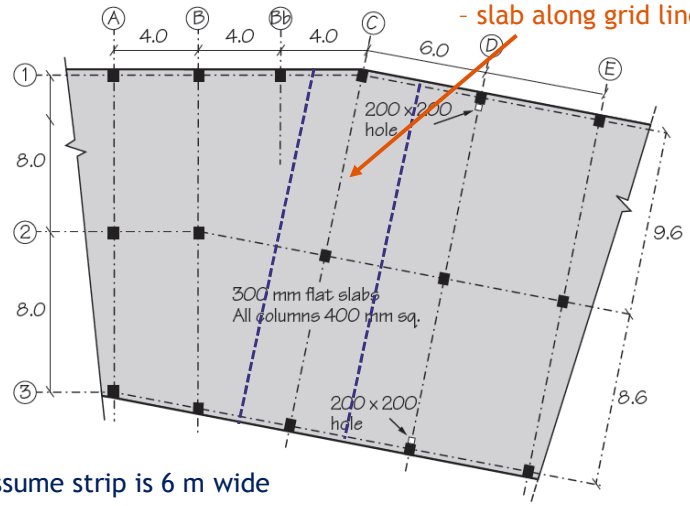


Design information

- Nominal cover = 30mm

Design strip along grid line C

Determine the reinforcement - slab along grid line C.



Assume strip is 6 m wide

Slab is 300 mm deep

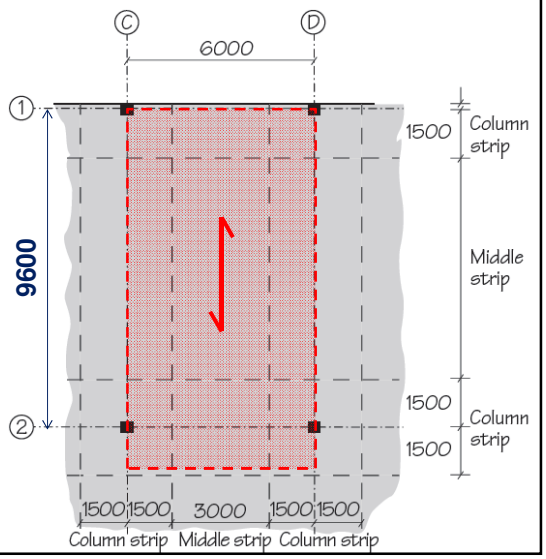
Flat slab
Worked example

For the previous flat slab example determine:

- Sagging reinforcement in the span 1-2

and

- Hogging reinforcement at support 2



Analysis

Actions:

g_k = 0.30 x 25 + 1.0 = 8.5 kN/m^2

q_k = 4.0 kN/m^2

n = 1.25 x 8.5 + 1.5 x 4.0 = 16.6 kN/m^2 NB. Exp(6.10b) used!

Analysis: using coefficients from Concise Table 15.3:
(Adjacent spans are 9.6 and 8.6 m. 8.6/9.6 = 0.89: i.e. > 85% so using coefficients is appropriate.)

Effective span = 9.6 - 2 x 0.4 + 2 x 0.3/2 = 9.5 m

In panel: sagging moment,

M_{Ed} = (1.25 x 8.5 x 0.09 + 1.5 x 4 x0.100) x 6.0 x 9.5^2 = 842.7 kNm

Along support 2: hogging moment

M_{Ed} = 16.6 x 0.106 x 6.0 x 9.5^2 = 952.8 kNm

↑ See Note to Concise Table 15.3 for support of 2-span slab

Division of moments

Location	Negative moments	Positive moments
Column strip	60% – 80%	50% – 70%
Middle strip	40% – 20%	50% – 30%

Notes
The total negative and positive moments to be resisted by the column and middle strips together should always add up to 100%

The distribution of design moments given in BS 8110 (column strip: hogging 75%, sagging 55%; middle strip: hogging 25%, sagging 45%) may be used

	M_{Ed}	
	Column strip	Middle strip
+ve sagging	0.50 x 842.7/3.0 = 140.5 kNm/m	0.50 x 842.7/3.0 = 140.5 kNm/m

(50% taken in both column and middle strips)

From analysis

3.4.5 Design grid line C

Effective depth, d:

$d = 300 - 30 - 20/2 = 260$ mm

a) **Flexure: column strip and middle strip, sagging**

$M_{Ed} = 140.5$ kNm/m

$K = M_{Ed}/bd^2f_{ck} = 140.5 \times 10^6/(1000 \times 260^2 \times 30) = 0.069$

$z/d = 0.94$ (Using Concise table 15.5)

$z = 0.94 \times 260 = 244$ mm

$A_s = M_{Ed}/f_{yd}z = 140.5 \times 10^6/(244 \times 500/1.15) = 1324$ mm^2/m

($\rho = 0.51\%$)

Try H20 @ 200 B1 (1570 mm^2/m)

Alternative

$z = d\,[\,1 + (1 - 3.529K)^{0.5}]/2 = 260[1 + (1 - 3.529 \times 0.069)^{0.5}]/2 = 243$ mm

Hogging Moments

	M_{Ed}	
	Column strip	Middle strip
-ve hogging	**0.70** x 952.8/3.0 = 222.3 kNm/m	**0.30** x 952.8/3.0 = 95.3 kNm/m

(70% taken in column strip and 30% in middle strip)

c) Flexure: column strip, hogging

M_{Ed} = 222.3 kNm/m

$K = M_{Ed}/bd^2f_{ck} = 222.3 \times 10^6/(1000 \times 260^2 \times 30) = 0.109$

$z/d = 0.89$ (Using Concise Table 15.5)

$z = 0.89 \times 260 = 231$ mm

$A_s = M_{Ed}/f_{yd}z = 222.3 \times 10^6/(231 \times 500/1.15) = 2213$ mm^2/m

Alternatively

$z = d\,[\,1 + (1 - 3.529K)^{0.5}]/2 = 260[1 + (1 - 3.529 \times 0.109)^{0.5}]/2 = 232$ mm

Hogging Moments

d) Flexure: middle strip, hogging

M_{Ed} = 95.3 kNm/m

$K = M_{Ed}/bd^2f_{ck} = 95.3 \times 10^6/(1000 \times 260^2 \times 30) =$ **0.047**

$z/d = 0.95$ (Using Concise table 15.5)

$z = 0.95 \times 260 = 247$ mm

$A_s = M_{Ed}/f_{yd}z = 95.3 \times 10^6/(247 \times 500/1.15) = 887$ mm^2/m

Alternatively

$z = d\,[\,1 + (1 - 3.529K)^{0.5}]/2$ $= 260[1 + (1 - 3.529 \times 0.047)^{0.5}]/2$

$= 248$ mm $\leq 0.95d \leq 247$ mm

$= 247$

Reinforcement distribution

Total area of reinforcement:

$A_{s,tot}$ = 2213 x 3 + 887 x 3 = 9300 mm^2

50% $A_{s,tot}$ = 9300/2 = 4650 mm^2

This is spread over a width of 1.5 m

$A_{s,req}$ = 4650/1.5 = 3100 mm^2/m

Use H20 @ 100 ctrs T(3140 mm^2/m)

Remaining column strip:

$A_{s,req}$ = (2213 x 3 - 4650)/1.5 = 1326 mm^2/m

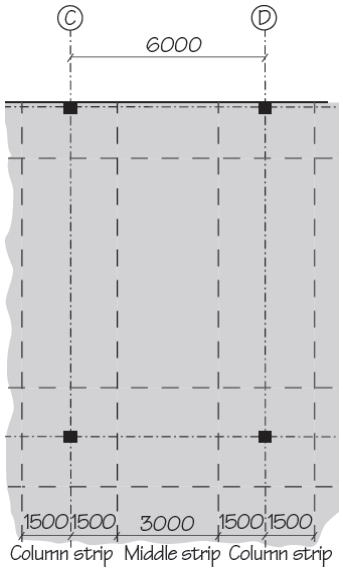
Use H20 @ 200 ctrs T(1570 mm^2/m)

Or use H16 @ 100 ctrs(1540 mm^2/m)

Middle strip: $A_{s,req}$ = 887 mm^2/m

Use H16 @ 200 ctrs T(1010mm^2/m)

Or use H12 @ 100 ctrs (1130mm^2/m)



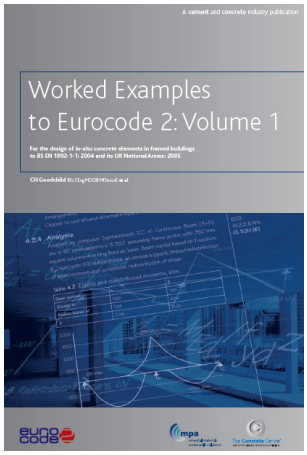
Exercise

Lecture 5

Check an edge column for punching shear

Punching shear Exercise

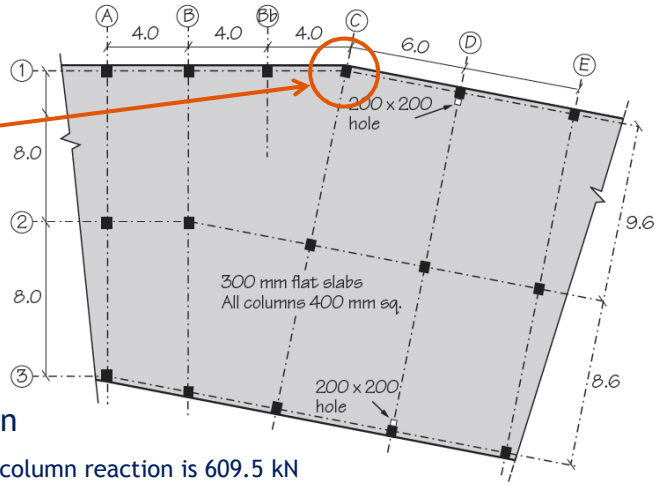
Based on the flat slab in section 3.4 of Worked Examples to EC2: Volume 1



Punching shear at column C1

400 mm Square Column

300 mm flat slab C30/37 concrete



Design information

- At C1 the ultimate column reaction is 609.5 kN
- Effective depths are 260mm & 240mm
- Reinforcement: ρ_{ly} = 0.0080, ρ_{lz} = 0.0069

Punching shear exercise

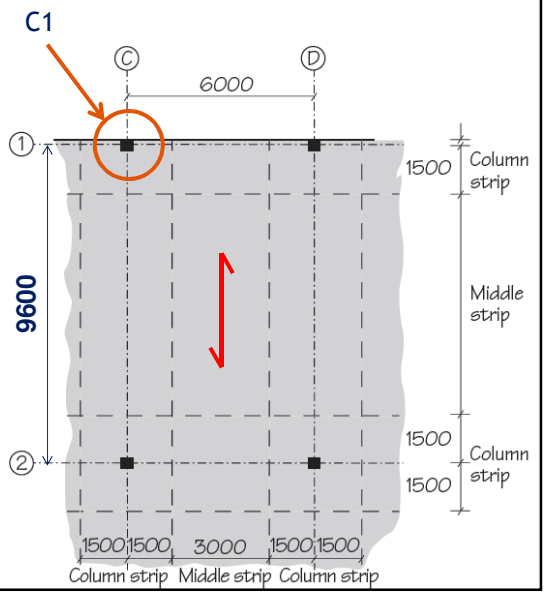
For the previous flat slab example:

a) Check the shear stress at the perimeter of column C1. The u_0 perimeter.

b) Check the shear stress at the basic perimeter, u_1.

c) Determine the distance of the u_{out} perimeter from the face of column C1.

d) Determine the area of shear reinforcement required on a perimeter. i.e. find A_{sw} for the u_1 perimeter.



Working space

Working space