Methods for checking deflection

This chapter describes the use of Eurocode 2 to check deflection by calculation. The alternative method for complying with the code requirements is to use the deemed-to-satisfy span-to-effective-depth ratios, which are appropriate and economic for the vast majority of designs. Further guidance on the span-to-effective-depth method is given in Chapters 3, 4 and 7, originally published as Beams, Slabs and Flat slabs. However, there are situations where direct calculation of deflection is necessary, as listed below:

- When an estimate of the deflection is required.
- When deflection limits of span/250 for quasi-permanent actions (see reference 5 for Eurocode terminology) or span/500 for partition and/or cladding loads are not appropriate.
- When the design requires a particularly shallow member, direct calculation of deflection may provide a more economic solution.
- To determine the effect on deflection of early striking of formwork or of temporary loading during construction.

Overview

In the past structures tended to be stiff with relatively short spans. As technology and practice have advanced, more flexible structures have resulted. There are a number of reasons for this, including:

- The increase in reinforcement strength leading to less reinforcement being required for the ultimate limit state (ULS) and resulting in higher service stresses in the reinforcement.
- Increases in concrete strength resulting from the need to improve both durability and construction time, and leading to concrete that is more stiff and with higher service stresses.

What affects deflection?

There are numerous factors that affect deflection. These factors are also often time-related and interdependent, which makes the prediction of deflection difficult.

The main factors are:

- Concrete tensile strength
- Creep
- Elastic modulus

Other factors include:

- Degree of restraint
- Magnitude of loading
- Time of loading
- Duration of loading
- Cracking of the concrete
- Shrinkage
- Ambient conditions
- Secondary load-paths
- Stiffening by other elements
How to design concrete structures using Eurocode 2

- A greater understanding of structural behaviour and the ability to analyse that behaviour quickly by computer.
- The requirement to produce economic designs for slabs whose thicknesses are typically determined by the serviceability limit state (SLS) and which constitute 80% to 90% of the superstructure costs.
- Client requirements for longer spans and greater operational flexibility from their structures.

Factors affecting deflection

An accurate assessment of deflection can only be achieved if consideration is given to the factors that affect it. The more important factors are discussed in detail below.

Tensile strength

The tensile strength of concrete is an important property because the slab will crack when the tensile stress in the extreme fibre is exceeded. In Eurocode 2 the concrete tensile strength, $f_{ctm}$, is a mean value (which is appropriate for deflection calculations) and increases as the compressive strength increases. This is an advancement when compared with BS 8110 where the tensile strength is fixed for all concrete strengths.

The degree of restraint to shrinkage movements will influence the effective tensile strength of the concrete. A layout of walls with high restraint will decrease the effective tensile strength. Typical examples of wall layouts are given in Figure 1. For a low restraint layout the following expression may be used for the concrete tensile strength:

$$f_{ctm,fl} = (1.6 - h/1000)f_{ctm} > f_{ctm}$$

where

- $f_{ctm,fl}$ = Mean flexural tensile strength of reinforced concrete
- $f_{ctm}$ = Mean tensile strength of concrete

Creep

Creep is the time-dependant increase in compressive strain in a concrete element under constant compressive stress. Creep is usually considered in the design by modifying the elastic modulus using a creep coefficient, $\varphi$, which depends on the age at loading, size of the member and ambient conditions, in particular relative humidity. Eurocode 2 gives advice on the calculation of creep coefficients in detail in Annex B. It also advises on the appropriate relative humidity to use in Figure 3.1.

Elastic modulus

The elastic modulus of concrete is influenced by aggregate type, workmanship and curing conditions. The effective elastic modulus under sustained loading will be reduced over time due to the effect of creep. These factors mean that some judgement is required to determine an appropriate elastic modulus. Eurocode 2 gives recommended values for the 28-day secant modulus, $E_{cm}$, (in Table 3.1) and makes recommendations for adjustments to these values to account for different types of aggregate. The long-term elastic modulus should be taken as:

Figure 1
Typical floor layouts

![Typical floor layouts](image-url)
Deflection calculations

Commercial pressures often lead to a requirement to strike the formwork as soon as possible and move on to subsequent floors, with the minimum of propping. Tests on flat slabs have demonstrated that as much as 70% of the loads from a newly cast floor (formwork, wet concrete, construction loads) may be carried by the suspended floor below. It can generally be assumed that early striking of formwork will not greatly affect the deflection after installing the cladding and/or partitions. This is because the deflection affecting partitions will be smaller if the slab becomes ‘cracked’ before, rather than after, the installation of the cladding and/or partitions.

Cracking

Deflection of concrete sections is closely linked to the extent of cracking and the degree to which cracking capacity is exceeded. The point at which cracking occurs is determined by the moments induced in the slab and the tensile strength of the concrete, which increases with age. Often the critical situation is when the slab is struck, or when the load of the slab above is applied. Once the slab has cracked its stiffness is permanently reduced.

It is therefore necessary to find the critical loading stage at which cracking first occurs. This critical loading stage corresponds with the minimum value of K, where:

\[ K = \frac{f_{cm}}{W \sqrt{0.5}} \]

where

\[ W = \text{The serviceability loading applied up to that stage} \]
\[ f_{cm} = \text{The concrete tensile strength at that stage} \]

Where the frequent combination is the critical load stage, then the degree of cracking (\( \phi \)) calculated for the frequent combination should also be used for the quasi-permanent combination, but not for

\[ \epsilon_{lt} = \epsilon_{cm}/(1 + \phi) \]

where

\[ \epsilon_{cm} = 28\text{-day tangent modulus} = 1.05 E_{cm} \]
\[ \phi = \text{Creep factor. (Note that with Eurocode 2, } \phi \text{ relates to a 28-day short-term elastic modulus, whereas a ‘true’ creep factor would be associated with the modulus at the age of loading.)} \]

The assessment of the long-term \( E \)-value can be carried out more accurately after the contractor has been appointed because they should be able to identify the concrete supplier (and hence the type of aggregates) and also the construction sequence (and hence the age at first loading).

**Loading sequence**

The loading sequence and timing may be critical in determining the deflection of a suspended slab because it will influence the point at which the slab will crack (if at all) and is used to calculate the creep factors for the slab. A loading sequence is shown in Figure 2, which shows that in the early stages relatively high loads are imposed while casting the slab above. The loading sequence may vary, depending on the construction method.

Smaller loads are imposed when further slabs are cast above. The loads are then increased permanently by the application of the floor finishes and erection of the partitions. Finally, the variable actions are applied to the structure and, for the purpose of deflection calculation, the quasi-permanent combination should be used. (See Chapter 1, originally published as *Introduction to Eurocodes* for further information on combinations of actions.) However, it is likely that the quasi-permanent combination will be exceeded during the lifetime of the building and, for the purpose of determining whether the slab might have cracked, the frequent combination may be critical.

**Figure 2**

Loading history for a slab – an example

![Loading history for a slab – an example](image)
Further information can be found in the best practice guide Early-striking and improved backpropping5.

Shrinkage curvature
Shrinkage depends on the water/cement ratio, relative humidity and the size and shape of the member. The effect of shrinkage in an asymmetrically reinforced section is to induce a curvature that can lead to significant deflection in shallow members. This effect should be considered in the deflection calculations.

Methods for calculating deflections
Two methods for calculating deflection are presented below, and these are based on the advice in TR58 Deflections in concrete slabs and beams8.

Rigorous method
The rigorous method for calculating deflections is the most appropriate method for determining a realistic estimate of deflection. However, it is only suitable for use with computer software. The Concrete Centre has produced a number of spreadsheets that use this method to carry out deflection calculations for a variety of slabs and beams8. These offer a cost-effective way to carry out detailed deflection calculations, and they include the ability to consider the effect of early age loading of the concrete. Figure 3 illustrates the principles of the method and shows how the factors affecting deflection are considered in the rigorous deflection calculations.

Finite element analysis may also be used to obtain estimates of deflection. In this case the principles in Figure 3 should be applied if credible results are to be obtained.

Panel 1
Determining long term elastic modulus of elasticity

Calculate long-term elastic modulus, $E_{LT}$ from:

$$E_{LT} = \sum \left( \frac{W_i}{E_{eff,i}} \right)$$

where

- $E_{eff,i}$ = $E_{cm} \left( 1 + \Phi_i \right)$
- $W_i$ = Serviceability load at stage $n$
- $\Phi_i$ = Creep coefficient at relevant loading time and duration

Overall deflection (quasi-permanent combination)
Deflection affecting partitions/cladding (Frequent combination deflection less deflection at time of installation)
8. Deflection calculations

Table 1
Concrete properties

<table>
<thead>
<tr>
<th>$f_{ck}$</th>
<th>MPa</th>
<th>20</th>
<th>25</th>
<th>28</th>
<th>30</th>
<th>32</th>
<th>35</th>
<th>40</th>
<th>50</th>
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<td>$f_{cm}$</td>
<td>MPa</td>
<td>28</td>
<td>33</td>
<td>36</td>
<td>38</td>
<td>40</td>
<td>43</td>
<td>48</td>
<td>58</td>
</tr>
<tr>
<td>$f_{cm} - (0.3 f_{ck}/3) \leq C50/60$</td>
<td>MPa</td>
<td>2.21</td>
<td>2.56</td>
<td>2.77</td>
<td>2.90</td>
<td>3.02</td>
<td>3.21</td>
<td>3.51</td>
<td>4.07</td>
</tr>
<tr>
<td>$f_{cm} - (0.3 f_{cm}/3) \leq C50/60$</td>
<td>MPa</td>
<td>2.77</td>
<td>3.09</td>
<td>3.27</td>
<td>3.39</td>
<td>3.51</td>
<td>3.68</td>
<td>3.96</td>
<td>4.50</td>
</tr>
<tr>
<td>$E_{cm} = (22 \sqrt{f_{cm}}/10) GPa$</td>
<td></td>
<td>30.0</td>
<td>31.5</td>
<td>32.3</td>
<td>32.8</td>
<td>33.3</td>
<td>34.1</td>
<td>35.8</td>
<td>37.3</td>
</tr>
<tr>
<td>$E_{c28} = (1.05 E_{cm}) GPa$</td>
<td></td>
<td>31.5</td>
<td>33.0</td>
<td>33.9</td>
<td>34.5</td>
<td>35.0</td>
<td>35.8</td>
<td>37.0</td>
<td>39.1</td>
</tr>
<tr>
<td>$e_{cd,0}$ CEM class R, RH = 50% microstrain</td>
<td></td>
<td>746</td>
<td>706</td>
<td>683</td>
<td>668</td>
<td>653</td>
<td>632</td>
<td>598</td>
<td>536</td>
</tr>
<tr>
<td>$e_{cd,0}$ CEM class R, RH = 80% microstrain</td>
<td></td>
<td>416</td>
<td>394</td>
<td>381</td>
<td>372</td>
<td>364</td>
<td>353</td>
<td>334</td>
<td>299</td>
</tr>
<tr>
<td>$e_{cd,0}$ CEM class N, RH = 50% microstrain</td>
<td></td>
<td>544</td>
<td>512</td>
<td>494</td>
<td>482</td>
<td>471</td>
<td>454</td>
<td>428</td>
<td>379</td>
</tr>
<tr>
<td>$e_{cd,0}$ CEM class N, RH = 80% microstrain</td>
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<td>303</td>
<td>286</td>
<td>275</td>
<td>269</td>
<td>263</td>
<td>253</td>
<td>239</td>
<td>212</td>
</tr>
<tr>
<td>$e_{cd,0}$ CEM class S, RH = 50% microstrain</td>
<td></td>
<td>441</td>
<td>413</td>
<td>397</td>
<td>387</td>
<td>377</td>
<td>363</td>
<td>340</td>
<td>298</td>
</tr>
<tr>
<td>$e_{cd,0}$ CEM class S, RH = 80% microstrain</td>
<td></td>
<td>246</td>
<td>230</td>
<td>221</td>
<td>216</td>
<td>210</td>
<td>202</td>
<td>189</td>
<td>166</td>
</tr>
</tbody>
</table>

Key
- $f_{cm}$ may be used when striking at less than 7 days or where construction overload is taken into account.

Panel 2
Useful Expressions for a rectangular section

\[
x_i = \left(\frac{bh^3}{6} + \left(\frac{b}{2} - x_i\right)^2\right) + \left(\frac{bh}{6} + \left(\frac{b}{2} - x_i\right)^2\right)
\]

\[
x_i = \left(\frac{bh}{6} + \left(\frac{b}{2} - x_i\right)^2\right) + \left(b - x_i\right)^2 + \left(\frac{bh}{6} + \left(\frac{b}{2} - x_i\right)^2\right)
\]

\[
x_i = \left(\frac{bh}{6} + \left(\frac{b}{2} - x_i\right)^2\right) + \left(b - x_i\right)^2 + \left(\frac{bh}{6} + \left(\frac{b}{2} - x_i\right)^2\right)
\]

\[
\frac{1}{\alpha} = \frac{\varepsilon_{cd} - \varepsilon_{cd,0}}{\varepsilon_{cd,0}} + \left(1 - \frac{1}{\alpha}\right) \frac{\varepsilon_{cd} - \varepsilon_{cd,0}}{\varepsilon_{cd,0}}
\]

where
- $A_t = $ Area of tension reinforcement
- $A_c = $ Area of compression reinforcement
- $b = $ Breadth of section
- $d = $ Effective depth to tension reinforcement
- $d_2 = $ Depth to compression reinforcement
- $h = $ Overall depth of section
- $\alpha = $ Modular ratio
- $S_t = A_t(d - x_t) - A_c(x_t - d_2)$
- $S_c = A_c(d - x_c) - A_t(x_c - d_2)$

Figure 4
Method for determining creep coefficient $\psi(t_0, t)$

Notes
1. $t_0 = $ age of concrete at time of loading
2. $h_0 = 2A_t/u$
3. Intersection point between lines D & E can also be above point A
4. For $t_0 > 100$ it is sufficiently accurate to assume $t = 100$
Simplified method
A simplified method for calculating deflection is presented in Figure 5. It is feasible to carry out these calculations by hand, and they could be used to roughly verify deflection results from computer software, or used where a computer is not available.

The major simplification is that the effects of early age loading are not considered explicitly, rather an allowance is made for their effect when calculating the cracking moment. Simplified creep factors are used and deflection from the curvature of the slab is approximated using a factor.

Figure 6
Values for K for various bending moment diagrams

<table>
<thead>
<tr>
<th>Loading</th>
<th>Bending moment diagram</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_c )</td>
<td>( M )</td>
<td>0.125</td>
</tr>
<tr>
<td>( M_c )</td>
<td>( M = Wa(1-a)l )</td>
<td>( \frac{3-4a^2}{48(1-a)} ) if ( a = \frac{1}{2} ), ( K = \frac{1}{12} )</td>
</tr>
<tr>
<td>( W/2 )</td>
<td>( M = Wa(1-a)l )</td>
<td>0.0625</td>
</tr>
<tr>
<td>( q )</td>
<td>( q )</td>
<td>0.104</td>
</tr>
<tr>
<td>( M_c )</td>
<td>( M_c = \frac{W}{2}l )</td>
<td>0.102</td>
</tr>
<tr>
<td>( q )</td>
<td>( q = \frac{q}{15.6} )</td>
<td></td>
</tr>
</tbody>
</table>

The moment, \( M_c \), due to quasi-permanent actions at the critical section (i.e. mid-span or at support for cantilever):

\[ M_c = \frac{9f_{ck}L^2}{8} \]

Obtain concrete properties, \( f_{ck} \) and \( E_{cr} \) from Table 1.

Calculate creep coefficient, \( \psi(t,c) \), using either Figure 4 or Annex B (in which case look-up \( f_{ck} \) in Table 1).

1. Calculate long term elastic modulus, \( E_{ctm} \) from:
   \[ E_{ctm} = E_{cr0}(1 + \psi(t,c)) \]
   2. Calculate effective modulus ratio, \( \alpha_c \) from:
   \[ \alpha_c = \frac{E_{cr}}{E_{ctm}} \]
   where \( E_{cr} \) is elastic modulus for reinforcement (200 GPa).

Calculate cracking moment, \( M_c \) from:

\[ M_c = \frac{0.9f_{ck}L^2}{b} \] (Note the factor 0.9 has been introduced into this method because the loading sequence is not considered)

Yes

Is \( M_c > M_{fp} \)?

Yes

Section is cracked

\[ \zeta = 1 - 0.5(M_c/M_{fp})^{1/2} \]

Calculate total curvature

\[ \frac{1}{r_{QP}} = \frac{M_c}{E_{ctm}I_c} + \left(1 - \zeta \right) \frac{M_c}{E_{cr}I_c} \]

Calculate quasi-permanent deflection from:

\[ \delta_{QP} = KL \frac{1}{r_{QP}} \]

where \( K \) can be obtained from Figure 6 and \( L \) is the span.

No

Section is uncracked

\[ \zeta = 0 \]

Calculate total curvature

\[ \frac{1}{r_{QP}} = \frac{1}{r_n} + \frac{1}{r_{cd}} \]

Calculate quasi-permanent deflection from:

\[ \delta_{QP} = KL \frac{1}{r_{QP}} \]

Finish

Do you need to calculate deflection due to cladding and partitions?

Yes

Recalculate the section properties, curvature and hence deflection, \( \Delta_{cd} \), using \( \psi(t,c) \) or equivalent instead of \( \psi(t,c) \).

4. The approximate deflection affecting cladding and partitions is

\[ \delta = \Delta_{QP} - \Delta_{cd} \]
8. Deflection calculations

Precamber

A slab or beam can be precambered to reduce the effect of deflection below the horizontal (see Figure 8). However, in practice too much precamber is generally used and the slab remains permanently cambered. This is because of the difficulty in accurately calculating deflection. A precamber of up to half the quasi-permanent combination deflection could be used, but a lower figure is recommended. Precamber does not reduce the deflections affecting partitions or cladding.

Flat slabs

Flat slabs are very popular and efficient floor systems. However, because they span in two directions, it can be difficult to calculate their deflection. TR58 gives several suitable methods for assessing flat slab deflection. Of these, a popular method is to take the average deflection of two parallel column strips and to add the deflection of the middle strip spanning orthogonally to get an approximation of the maximum deflection in the centre of the slab.

The recommended acceptance criteria for a flat slab are shown in Figure 9.

Accuracy

The calculation of deflection in Eurocode 2 using the rigorous method presented here is more advanced than that in BS 8110. It can be used to take account of early-age construction loading by considering reduced early concrete tensile strengths.

However, the following influences on deflections cannot be accurately assessed:

- Tensile strength, which determines the cracking moment.
- Construction loading.
- Elastic modulus.

Therefore any calculation of deflection is only an estimate, and even the most sophisticated analysis can still result in +15% to -30% error. It is advisable to give a suitable caveat with any estimate of deflection that others are relying on.

---

**Table 2**

Values for $K_h$

<table>
<thead>
<tr>
<th>$h_0$ (mm)</th>
<th>$k_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.0</td>
</tr>
<tr>
<td>200</td>
<td>0.85</td>
</tr>
<tr>
<td>300</td>
<td>0.75</td>
</tr>
<tr>
<td>&gt;500</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Notes

$h_0$ is the notional size (mm) of the cross-section = $2A_c/u$

where

$A_c$ = Concrete cross-sectional area

$u$ = Perimeter of that part of the cross section which is exposed to drying

---

**Figure 7**

Coefficient for development of creep with time after loading

**Figure 8**

Precambering of slabs

**Figure 9**

Recommended acceptance criteria for flat slabs

Notes

If maximum permitted $\Delta = L/n$ and $X$ is the position of maximum $\Delta$

where

$L$ = Span

$n$ = Limiting span-to-depth ratio, e.g. 250

then the deflection at $X$ should not be greater than $2a/n$.

(Maximum deflection on gridlines may be more critical.)
Cladding tolerances

Deflection may affect cladding or glazing in the following ways:

- When a slab deflects, the load on the central fixings will be relieved and shed to outer fixings.
- Manufacturers may say that their glazed systems can only accommodate deflection as low as 5 mm.

There should be open discussions between the designers for the various elements to determine the most cost-effective way of dealing with the interaction of the structure and cladding.

References

5. NARAYANAN, R S & BROOKER, O. How to design concrete structures using Eurocode 2: Introduction to Eurocodes. The Concrete Centre, 2005